

## **Course Material Bundle of Nuclear Physics - I**

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### **Topics to be covered**

1. History of Nuclear Physics
2. Nuclear Size, Mass, Charge & density
3. Packing fraction
2. Classification of The Nuclei
3. Packing Fraction
4. Nuclear Stability Related To The Neutron-Proton Ratio
5. Binding Energy And Nuclear Stability
6. Mass Defect Binding Energy, Its Types, Description, Bulk Properties
7. Nuclear forces
8. Properties of Nuclear Forces
9. Meson Theory of Nuclear Forces
10. The Interaction of Radiation with Matter
11. Gas-filled detectors (ionization chamber )
12. Linear accelerator
13. Cyclotron
14. Betatron

Text Book: Kaplan, I. (1980) Nuclear Physics, Addison Wesley Publishing Company.

### **Reference Books:**

1. Littlefield, T. A. & Thorley, N. (1979) Atomic and Nuclear Physics- An Introduction, 3rd edition, Van Norstrand Reinhold.
2. Wong, S.S.M. (1990) Introductory Nuclear Physics, Prentice Hall Publisher.
3. Burge, E.T. (1988) Atomic Nuclei and their Particles, Oxford University Press.
4. Blin, R.J. (1991) Nuclear and Particle Physics, Chapman & Hall London.
5. Nuclear And Particle Physics by CL Arora

**Nuclear physics:** The branch of physics that deals with the constituents of nucleus and interactions of atomic nuclei.

It includes studies of nuclear components such as protons and neutrons, forces such as the strong force, and phenomena such as radioactive decay, nuclear fission, and nuclear fusion.

The most commonly known applications of nuclear physics are:

1. Nuclear power generation
2. Nuclear weapons technology
3. Nuclear medicine and magnetic resonance imaging
4. Ion implantation in materials engineering
5. Radiometric dating, radioactive dating or radioisotope dating e.g. Radiocarbon dating in geology and archaeology
6. Nuclear Forensics

### **History of Nuclear Physics**

**1895:** Wilhelm Roentgen of Germany, while conducting experiments with cathode rays, accidentally discovers a new and different kind of ray. These rays were so mysterious that Roentgen named them "x-rays." He received the first Nobel Prize in Physics in 1901 for this discovery.

**1896:** French physicist Antoine Henri Becquerel's experiments led to the discovery of radioactivity. He observed that the element uranium can blacken a photographic plate, even though separated from it by glass or black paper. He also observed that the rays that produce the darkening are capable of discharging an electroscope, indicating that the rays possess an electric charge.

**1897:** J. J. Thomson of Britain discovers the electron, while also studying cathode rays. He received the Nobel Prize in Physics in 1906 for this discovery.

**1899:** Ernest Rutherford discovers two kinds of rays emitting from radium. The first he calls alpha rays; the more penetrating rays he calls beta rays.

**1900:** Frederick Soddy observes spontaneous disintegration of radioactive elements into variants he calls "isotopes."

**1902:** Ernest Rutherford and Soddy publish theory of radioactive decay.

**1903:** Becquerel shares Nobel Prize for Physics with Pierre and Marie Curie for 1896 discovery of natural radioactivity.

**1904:** Rutherford discovers that alpha rays are heavy positively charged particles. In 1908, he is awarded a Nobel Prize in Chemistry for his work.

J. J. Thomson proposes the "plum-pudding" model of the atom. In it the atom is envisioned as electrons surrounded by a soup of positive charge, like plums surrounded by pudding.

**1905:** Albert Einstein publishes the special theory of relativity regarding convertibility of matter and energy ( $E=mc^2$ ).

**1911:** The "plum-pudding" is disproved by the gold foil experiment by Ernest Rutherford, when he discovered the nucleus of the atom.

Marie Curie receives a second Nobel Prize, this time in Chemistry, for the isolation of radium and polonium and for her investigation of their chemical properties.

**1913:** Niels Bohr publishes theory of atomic structure, combining nuclear theory with quantum theory.

**1915:** Albert Einstein publishes the general theory of relativity. The theory proposes that gravity, as well as motion, can affect the intervals of time and of space.

**1919:** Rutherford bombards nitrogen gas with alpha particles and obtains atoms of an oxygen isotope and protons. This transmutation of nitrogen into oxygen was the first artificially induced nuclear reaction.

**1920:** Rutherford speculates on the existence of the neutron at a lecture to the Royal Society of London.

**1925:** Werner Heisenberg, Max Born and later Erwin Schrödinger formulate quantum mechanics. Heisenberg is awarded the Nobel Prize in Physics in 1932 for the creation of quantum mechanics.

**1927:** Werner Heisenberg states the uncertainty principle, which states that it is not possible to simultaneously determine the position and momentum of a particle.

**1929:** Ernest O. Lawrence conceives idea for the first cyclotron, a device that greatly increased the speed with which protons could be hurled at atomic nuclei. He was awarded the 1939 Nobel Prize in Physics for the invention and development of the cyclotron and for results obtained with it.

John Cockcroft and E. T. S. Walton develop a high-voltage apparatus ("linear accelerator") for accelerating protons. With this they study nuclear reactions (atomic transmutation) and are awarded the 1951 Nobel Prize in Physics.

**1931:** Harold Urey discovers deuterium, an isotope of hydrogen that contains one proton and one neutron.

**1932:** James Chadwick discovers the neutron.

**Thomson Plum Pudding Model:** The view of the atom at Thomson's time was that it consisted of two components, with positive and negative electric charges, the latter now being the electrons. Thomson suggested a model where the electrons were embedded and free to move in a region of positive charge filling the entire volume of the atom – the so-called 'plum pudding model'.

**Achievement of the Model:** Plum pudding model' was able to account for the stability of atoms.

**Limitations of the Model:** 1. It failed to account for the discrete wavelengths observed in the spectra of light emitted from excited atoms.

2. It failed to explain the results of a classic series of experiments started in 1911 by Rutherford and continued by his collaborators, Geiger and Marsden.

### **Properties of Nucleus.**

**(i) Charge.** If  $Z$  is the charge number of nucleus i.e. the number of proton in it, then

Charge on the nucleus =  $+Ze$

Where  $e$  is the positive charge equal to the charge on the electron =  $1.6 \times 10^{-19}$  *Coulomb*.

**(ii) Mass.** If  $A$  is the mass number i.e. the total number of nucleons in the nucleus i.e.  $Z$  protons and  $(A-Z)$  neutrons, then the mass of the nucleus is very nearly equal to  $A$  atomic mass units. In terms of atomic units, the mass of carbon  $C^{12}$  is taken to be = 12 (a.m.u) and 1 a.m.u ( also written as 1u ) =  $1.6604 \times 10^{-27}$  kg = 931.48 MeV.

**(iii) Radius.** As the nucleus is approximately spherical, its volume is proportional to the total number of nucleons in it or its mass number  $A$ .

$$\therefore \frac{4}{3}\pi r^3 \propto A$$

Where  $r$  is the radius of the nucleus.

Hence  $r \propto A^{1/3}$

Or  $r = r_0 A^{1/3}$

Where  $r_0 = 1.3 \times 10^{-15} \text{ m} = 1.3 \text{ fermi}$  [ 1 fermi (fm) =  $10^{-15} \text{ m}$  ]

**(iv) Density.**

Nuclear density  $\rho_N = \frac{\text{Nuclear mass}}{\text{Nuclear volume}}$

Nuclear mass =  $A m_N$  where  $A$  = mass number

and  $m_N$  = mass of the nucleus =  $1.67 \times 10^{-27} \text{ kg}$

$$\text{Nuclear volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A$$

$$\rho_N = \frac{A m_N}{\frac{4}{3}\pi r_0^3 A} = \frac{m_N}{\frac{4}{3}\pi r_0^3} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi (1.3 \times 10^{-15})^3} = 1.816 \times 10^{17} \text{ kg m}^{-3}$$

This shows that the nuclear matter is in a highly compressed state.

*As the density of the nucleus is independent of  $A$ , its value is almost the same for all nucleus.*

**(v) Binding energy.** The nucleons exert short range nuclear forces of attraction on each other. When any two particles attract each other, the sum of their masses, when separated, exceeds that of the bound system since energy (or mass) must be added to the system to separate it into component particles.

This energy is called *binding energy* and hold the nucleons together to form a stable nucleus. This stability of the nucleus is due to the decrease in mass of constituent particles which combine to form the nucleus.

For a nucleus having  $Z$  protons and  $N = (A - Z)$  neutrons;

**Assumed nuclear mass** =  $Zm_p + Nm_n$  Where  $m_p$  is the mass of the proton and  $m_n$  that of the neutron.

**The real nuclear mass  $M_N$**  is however, less than the assumed nuclear mass.

The difference in assumed and real mass  $(Zm_p + Nm_n) - M_N = \Delta m$  gives decrease in mass and binding energy is given by

$$\text{Binding energy B.E.} = c^2 [(Zm_p + Nm_n) - M_N]$$

**Classification of the nuclei**

**Isotope:** Atoms with the same atomic number but different mass numbers are called isotopes e.g.  $^{11}_6\text{C}$ ,  $^{12}_6\text{C}$ ,  $^{13}_6\text{C}$

**Isotone:** Atoms with the same number of neutrons, but different numbers of protons are called isotones e.g.  $^9_3\text{Li}$ ,  $^{10}_4\text{Be}$ ,  $^{11}_5\text{B}$ ,  $^{12}_6\text{C}$

**Isobar:** Atoms with the same number of nucleons but different numbers of protons are called isobars e.g.  $^{12}_5\text{B}$ ,  $^{12}_6\text{C}$ ,  $^{12}_7\text{N}$

**Isomer:** Atoms with the same atomic and mass numbers but which are in different energy states are called nuclear isomers ( $\text{Tc}^{99\text{m}}$ ).

**Mirror nuclei:** Mirror nuclei have the same number of nucleons but the number of protons and neutrons are interchanged. e.g

Boron-11 and Carbon-11:  $^{11}_5\text{B}$   $^{11}_6\text{C}$



Carbon-13 and Nitrogen-13:  $^{13}\text{C}_6$  &  $^{13}\text{N}_7$

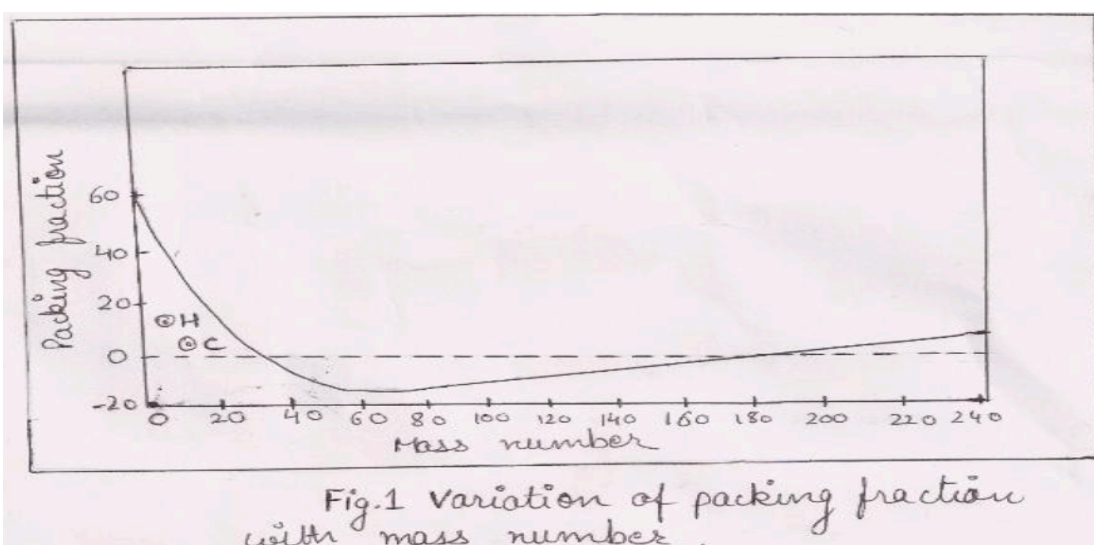
**Packing fraction:**

The ratio between mass defect and the mass number is called the packing fraction. Packing fraction means mass defect per nucleon.

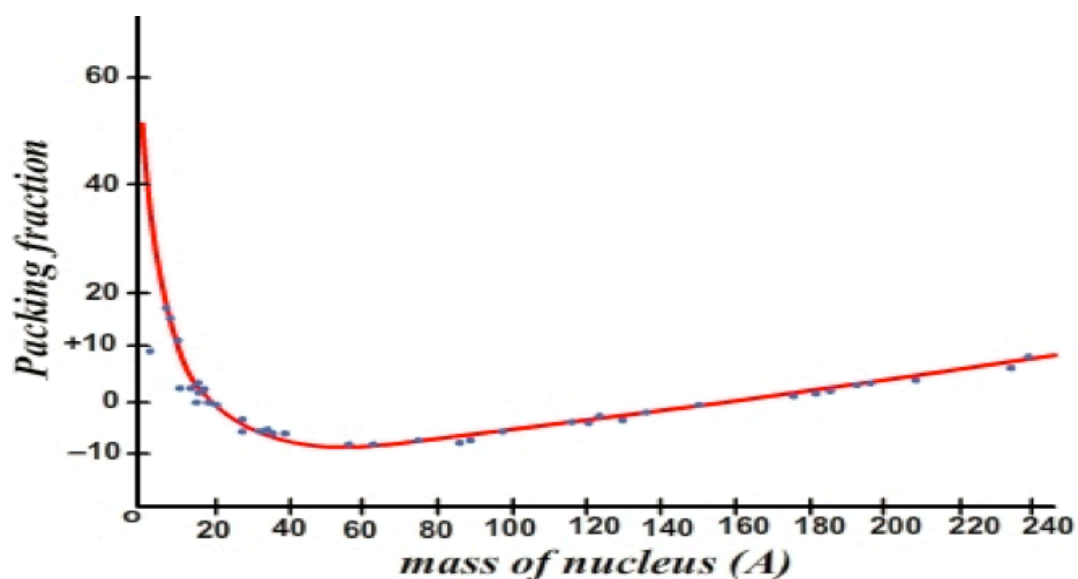
$$\text{Packing Fraction } f = \frac{M - A}{A}$$

- Packing fraction is used to measure the comparative stability of the atom.
- Packing fraction can have positive or can have negative sign.
- A positive packing fraction describes a tendency towards instability.
- A negative packing fraction means isotopic mass is less than actual mass number.

1.



2.



It can be seen that

1. The packing fraction is positive for elements having mass number below 20.
2. The packing fraction is negative for elements having mass number between 20 and 200.
3. The packing fraction is positive for elements having mass number greater than 200.
4. The packing fraction is zero for carbon  $A=12$  and  $Z=6$ . This does not mean that its B.E. is zero.
5. The mass defect and packing fraction only show their relations with respect to carbon.

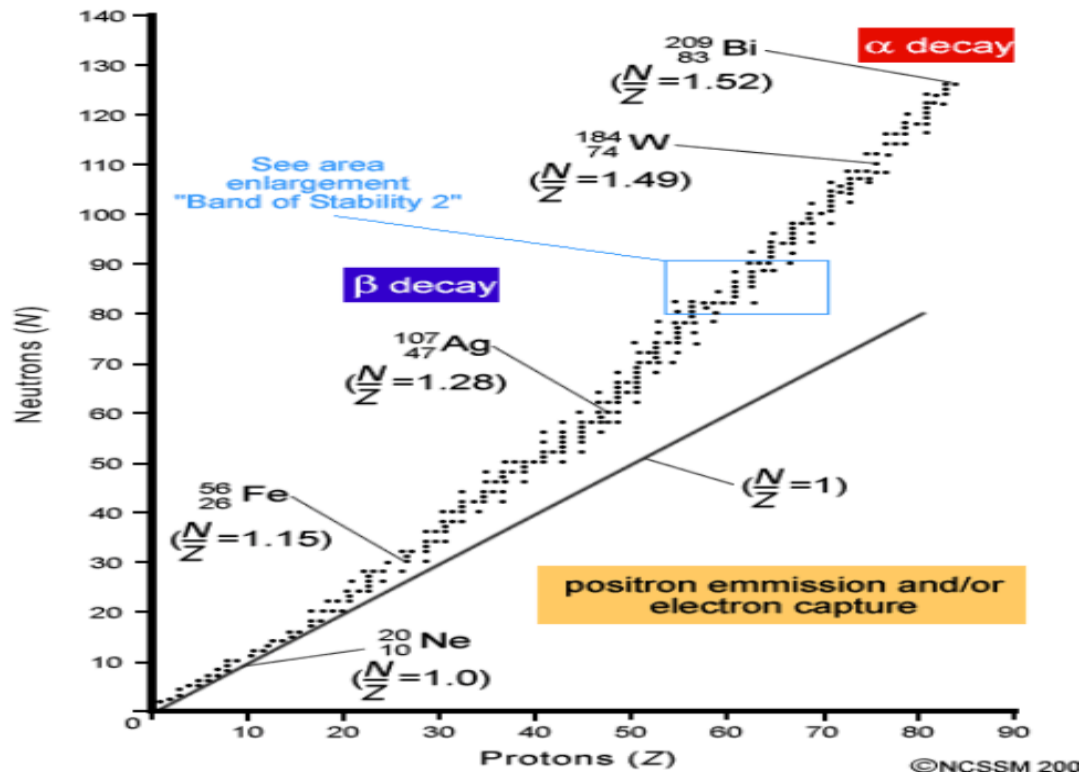
### **Even Odd theory of Nuclear Stability**

Z	N	Number of Stable Nuclides
Even	Even	165
Even	Odd	55
Odd	Even	50
Odd	Odd	5

**Question: How is nuclear stability related to the neutron-proton ratio?**

**Answer:**

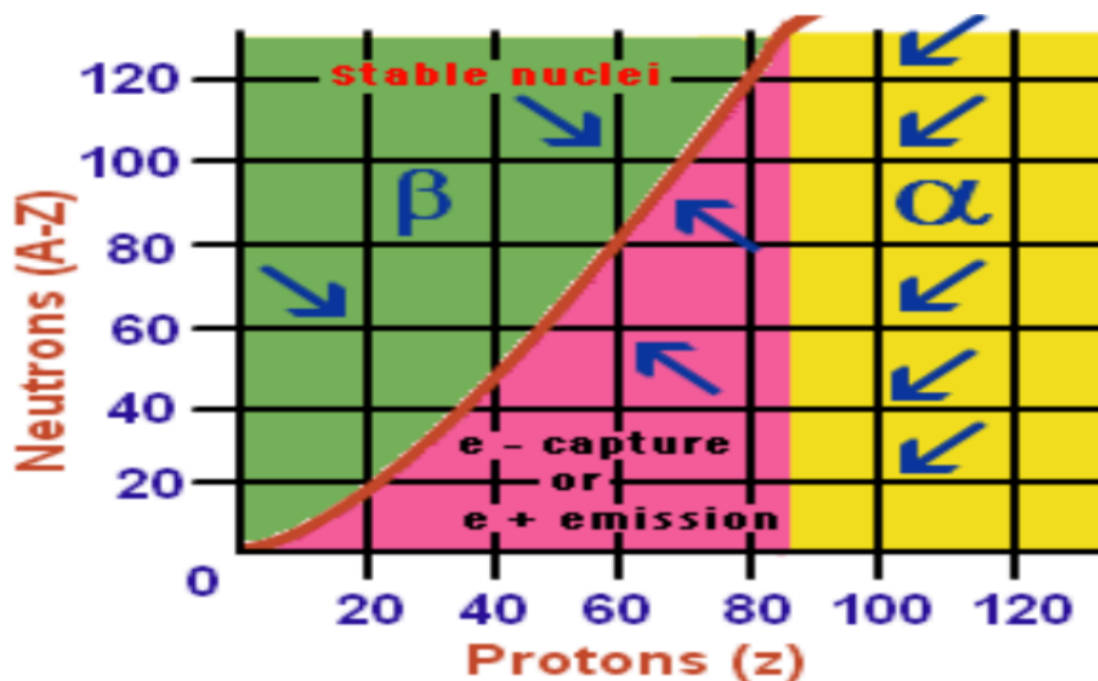
- The nucleus is unstable if the neutron-proton ratio is less than 1:1 or greater than 1.5:1
- At close distances, a strong nuclear force exists between nucleons. This attractive force comes from the neutrons.
- More protons in the nucleus need more neutrons to bind the nucleus together.
- The graph below is a plot of the number of neutrons versus the number of protons in various stable isotopes.
- The stable nuclei are in the pink band known as the belt of stability.
- They have a neutron/proton ratio between 1:1 and 1.5:1

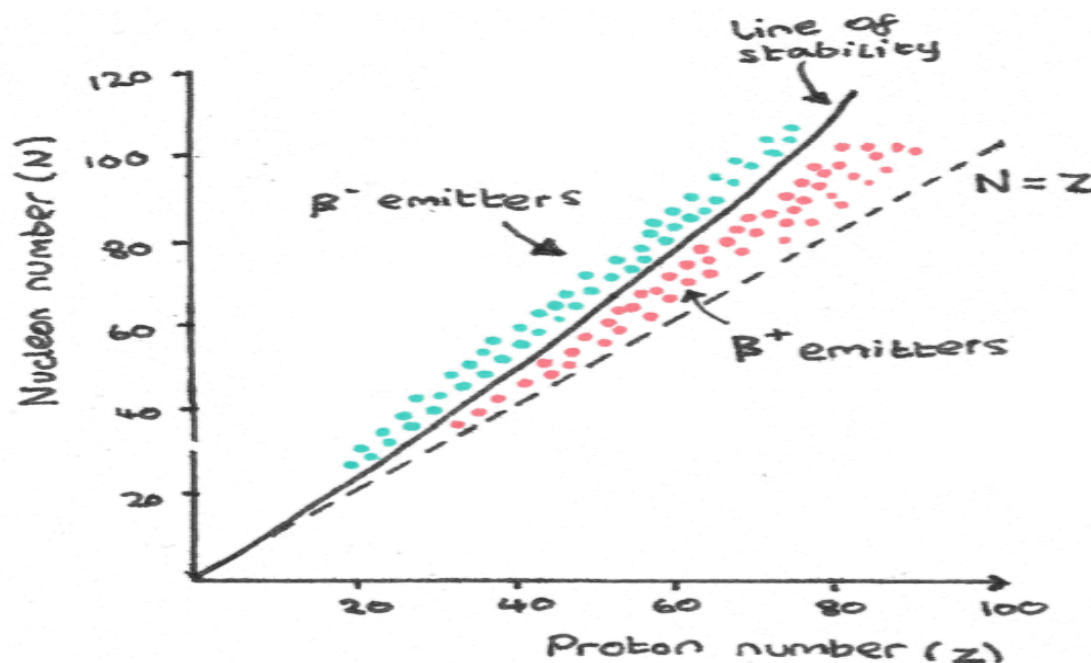


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- As the nucleus gets bigger, the electrostatic repulsions between the protons gets weaker.
- The nuclear strong force is about 100 times as strong as the electrostatic repulsions.
- It operates over only short distances.
- After a certain size, the strong force is not able to hold the nucleus together.
- Adding extra neutrons increases the space between the protons.
- This decreases their repulsions but, if there are too many neutrons, the nucleus is again out of balance and decays.

### Indicators of Nuclear Stability. Is it stable or unstable?

Is $Z > 83$ ?	If so, it is an alpha-emitter
Calculate the # of protons and neutrons, is it an even #?	If either are an even # there is a good chance it is STABLE.
Are there a MAGIC # of protons or neutrons?	2, 8, 20, 28, 50, 82, 114P, 126N, 184N-they're STABLE
Calculate the N/Z ratio	If you get to this step, then use the belt of stability to figure out.





### Question 1.12 :

Explain the terms mass defect, missing energy and binding energy of the nucleus and binding energy per nucleon.

**Answer:**

#### Mass defect:

The nucleus is formed by bringing protons and neutrons together. The mass of nucleus so formed is less than the sum of masses of the constituent protons and neutrons. This mass difference is called mass defect and is denoted by  $\Delta m$ . If  $Z$  is number of protons in the nucleus, then number of neutrons in the nucleus is  $(A-Z)$ . If  $m_p$  is mass of protons and  $m_n$  that of neutron, then

Sum of masses of protons and neutrons  $= Zm_p + (A-Z)m_n$

If  $M_N$  is actual mass of nucleus then, Mass defect  $= \Delta m = Zm_p + (A-Z)m_n - M_N$

Mass defect is defined as the difference between the sum of rest masses of nucleons forming the nucleus and actual rest mass of the nucleus. For example in case of deuteron which contain one proton and one neutron the combined mass is  $[1.0073 + 1.0087] = 2.0160$  a.m.u. where actual mass of deuteron nucleus is 2.0136 a.m.u.

Mass defect  $= \Delta m = 2.0160 - 2.0136 = 0.0024$  a.m.u.

#### Binding energy:

The mass of any permanently stable nucleus is found to be less than sum of masses of neutrons and protons which it contains. The fact is accounted for by conversion of part of mass energy of particle into energy of binding, the relation between the change in mass and binding energy being given by Einstein's equation;

Binding energy  $= E_B = \Delta mc^2$

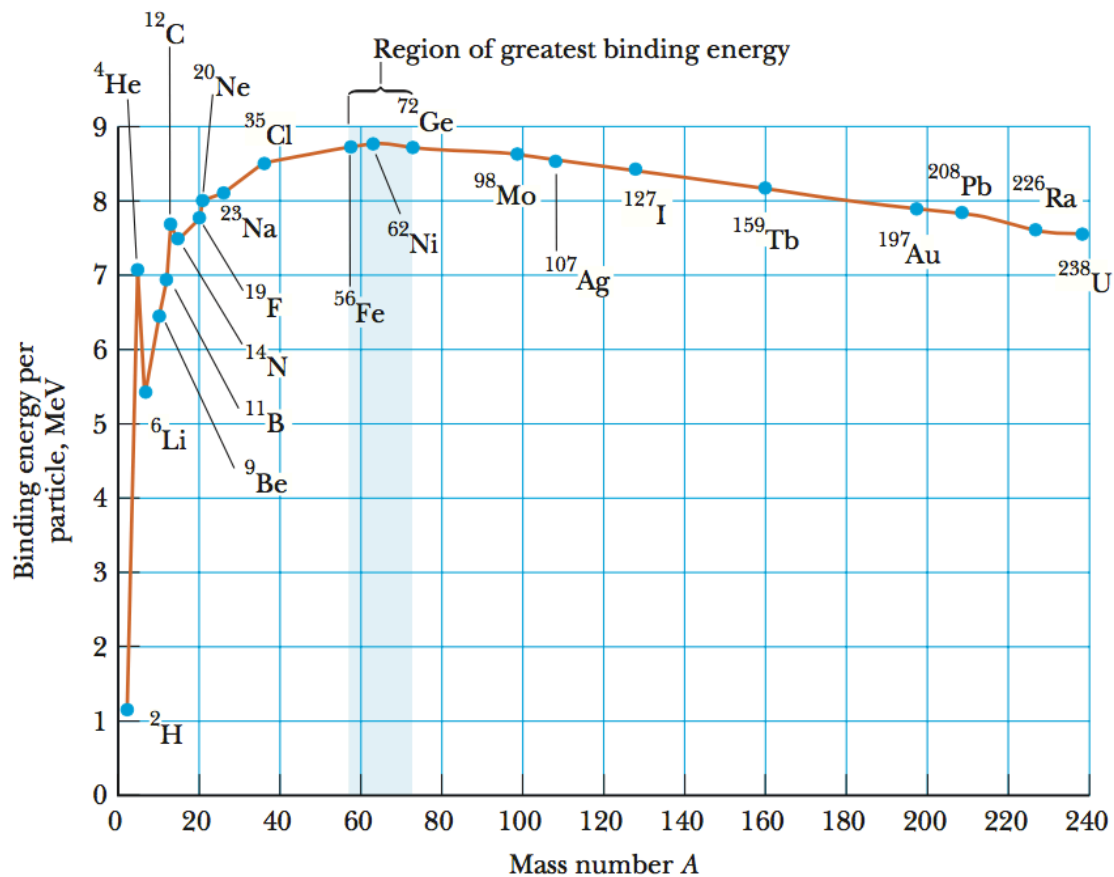


Figure 2: Binding energy per nucleon versus mass number for nuclei that lie along the line of stability. Some representative nuclei appear as blue dots with labels. (Nuclei to the right of  $^{208}\text{Pb}$  are unstable. The curve represents the binding energy for the most stable isotopes.)

The total binding energy for all atoms except the lighter atoms is given approximately by the empirical relation

$$E_B = 15.6Z \text{ MeV}$$

The binding energy of nucleus with charge  $Z$  and mass number  $A$  is given by,

$$E_B = c^2 [Zm_p + (A-Z)m_n - M_N]$$

Where  $M_N$  is mass of nucleus,  $m_p$  is mass of proton and  $m_n$  is mass of neutron.

The nucleus mass is therefor, given by

$$M_N = Zm_p + (A-Z)m_n - E_B/c^2$$

And atomic mass

$$M = Zm_e + M_N = Zm_e + Zm_p + (A-Z)m_n - E_B/c^2$$

$$= ZM_H + (A-Z)m_n - E_B/c^2$$

Where  $m_e$  is the mass of electron and  $M_H$  is the mass of hydrogen atom. The term  $E_B/c^2$  represents the mass equivalent of total binding energy i.e the energy which must be added to the nucleus in order to break it up into  $Z$  protons and  $(A-Z)$  neutrons.

Binding energy of the nucleus is therefore the energy which must be supplied to nucleus to break it into its constituent nucleons. The experimental values of BE varies from 2.23 MeV for deuteron, the lightest stable atom containing more than one nucleon to 1640 MeV for the heaviest stable nucleus  $^{209}\text{Bi}_{83}$ .

**Average binding energy :**

The average binding energy is binding energy per nucleon. It is energy required to release a nucleon from nucleus.

$$E_B/A = c^2 [Zm_p + (A-Z)m_n - M_N] / A$$

### **Binding energy and nuclear stability :**

The binding energy of nucleus give quantitative measure of its stability.

Consider the nuclide  ${}^2_2\text{He}^4$  which has BE about 28 MeV of energy. In other words the energy required to break up  ${}^2_2\text{He}^4$  nucleus in its constituent nucleons we must spend about 28 MeV of energy. Thus binding energy per nucleon is about 7 MeV. Greater the binding energy, more stable is the nucleus.

We also find for the nuclides like  ${}^2_2\text{He}^4$ ,  ${}^4_2\text{Be}^8$ ,  ${}^{12}_6\text{C}^{12}$ ,  ${}^{16}_8\text{O}^{16}$ ,  ${}^{20}_{10}\text{Ne}^{20}$  and  ${}^{24}_{12}\text{Mg}^{24}$  there is a sharp increase in binding energy. For these nuclides  $A=4n$  where  $n = 1, 2, 3, 4 \dots$  etc. All these nuclides can be built up from alpha-particles which is very highly bound system.

*There is a broad maximum in mass number range 40 to 120 at an average value of 8.5 MeV. The maximum value of binding energy per nucleon is  $\approx 8.8$  MeV for mass number  $A$  very nearly equal to 60. Thus binding energy per nucleon fall slowly to 7.4 MeV.*

The energy released in fission is several million times that released in chemical changes, binding energy of nucleus in process of fission is much higher than binding energy of electrons in chemical process. If two very light nuclei are made to fuse into single nucleus, a large amount of energy will be released. This is known as nuclear fusion.

Nuclides beyond  $A=238$  will have even smaller binding energy per nucleon and hence are less stable.

### **Release of nuclear energy in fission and fusion:**

From graph we see that value of binding energy per nucleon is very small both for light and very high nuclei so that these substances are unstable. The nuclei of intermediate masses (40 to 120) are most stable and very high amount of energy has to be supplied to liberate each of their nucleons.

It therefore follows, when a heavy nucleus breaks into lighter ones (nuclear fission) the end products formed have higher value of binding energy hereby resulting in liberation of energy. Similarly when two very light nuclei combine to form heavy nucleus (nuclear fusion) the high binding energy per nucleon of latter again results in liberation of energy.

### **Binding energy:**

Binding energy is the energy required to disassemble a whole system into separate parts. A bound system typically has a lower potential energy than the sum of its constituent parts; this is what keeps the system together. Often this means that energy is released upon the creation of a bound state.

### **Types of binding energy**

#### **Gravitational binding energy:**

Gravitational binding energy is the energy it takes to separate two objects that are bound to each other by gravity.

#### **Atomic Binding Energy:**

It is the energy required to disassemble an atom into free electrons and a nucleus. Electron binding energy is a measure of the energy required to free electrons from their atomic orbits.

### **Nuclear Binding Energy:**

At the nuclear level, nuclear binding energy is the energy required to disassemble a nucleus into the free, unbound neutrons and protons it is composed of. It is the energy equivalent of the mass defect, the difference between the mass number of a nucleus and its true measured mass.

### **Quantum Chromodynamics Binding Energy:**

Quantum chromodynamics binding energy is the energy which binds the various quarks together inside a hadron. (*quantum chromodynamics (QCD) is the theory of the strong interaction between quarks and gluons, the fundamental particles that make up composite hadrons such as proton, neutron and pion.*)

### **Bulk Properties of Binding Energy:**

1. For most nuclei, binding energy per nucleon is about 8 MeV.
2. BE is less for light nuclei but there are peaks for  $A$  in multiples of 4. (But note that the peak for  ${}^8\text{Be}$  is slightly lower than that for  ${}^4\text{He}$ . What does this imply?)
3. The most stable nuclei ( ${}^{62}\text{Ni}$  and  ${}^{64}\text{Fe}$ ) are in the region  $A \approx 60$ .
4. Light nuclei can gain binding energy per nucleon by fusing; heavy nuclei by fissioning.
5. The decrease in binding energy per nucleon for value of  $A > 60$  can be ascribed to the repulsion between the (charged) protons in the nucleus.

### **Nuclear Forces**

The radius  $r_n$  of the nucleus of mass number  $A$  is given by  $r = r_0 A^{1/3}$

where  $r_0$  is constant for all nuclei and has value  $1.3 \times 10^{-15}$  m. Hence the nuclear mass and nuclear volume are proportional to  $A$ . Therefore, the nuclear density is nearly constant for all nuclides.

The nucleus consists of protons and neutrons. As the protons carry positive charge, the electrostatic force of repulsion between them cause disruption of nucleus. The gravitational force of attraction between neutrons is too weak for observed binding energy of nuclei. Thus there must be some forces that bind the nucleons.

The nuclear force (or nucleon–nucleon interaction or residual strong force) is a force that acts between the protons and neutrons of atoms. Neutrons and protons, both nucleons, are affected by the nuclear force almost identically. Since protons have charge  $+1 e$ , they experience an electric force that tends to push them apart, but at short range the attractive nuclear force is strong enough to overcome the electromagnetic force. The nuclear force binds nucleons into atomic nuclei.

The nuclear force is powerfully attractive between nucleons at distances of about 1 femtometre (fm, or  $1.0 \times 10^{-15}$  metres), but it rapidly decreases to insignificance at distances beyond about 2.5 fm. At distances about 0.5 fm ~~less than 0.7 fm~~, the nuclear force becomes repulsive. This repulsive component is responsible for the physical size of nuclei, since the nucleons can come no closer than the force allows.

### **There are three types of nuclear forces:**

- (1) Force b/w two protons (p – p force)
- (2) Force b/w a proton and neutron (p - n force)
- (3) Force b/w two neutrons (n – n force)



**Here we discuss all of them:-**

(1)(p-p forces):

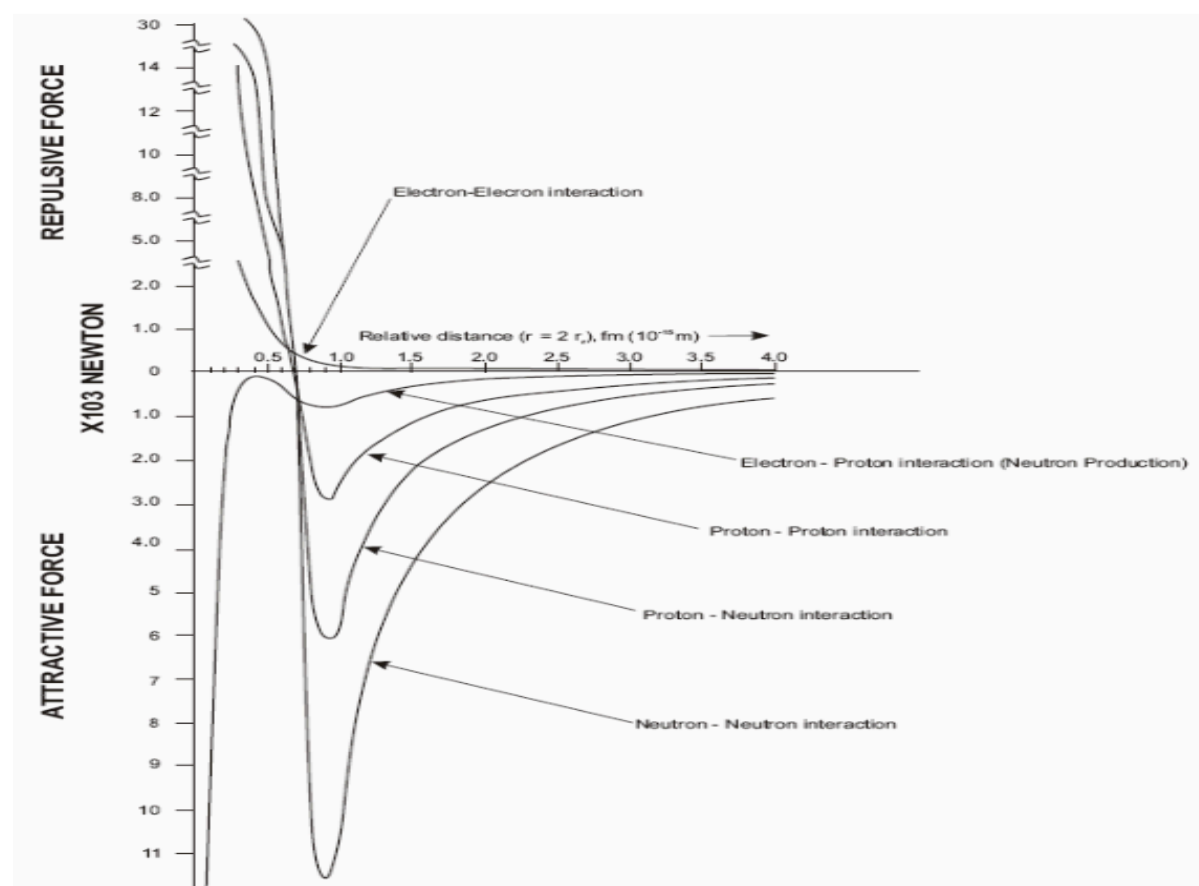
The (p-p) force is examined by proton-proton scattering experiment and shown in Figure . At large distances of separation protons repel one another by Coulomb electrostatic force. At distance of 3fm, a fairly sharp break in potential curve occurs. At smaller distances protons strongly attracts each other and this strong attractive force is nuclear force between a pair of protons.

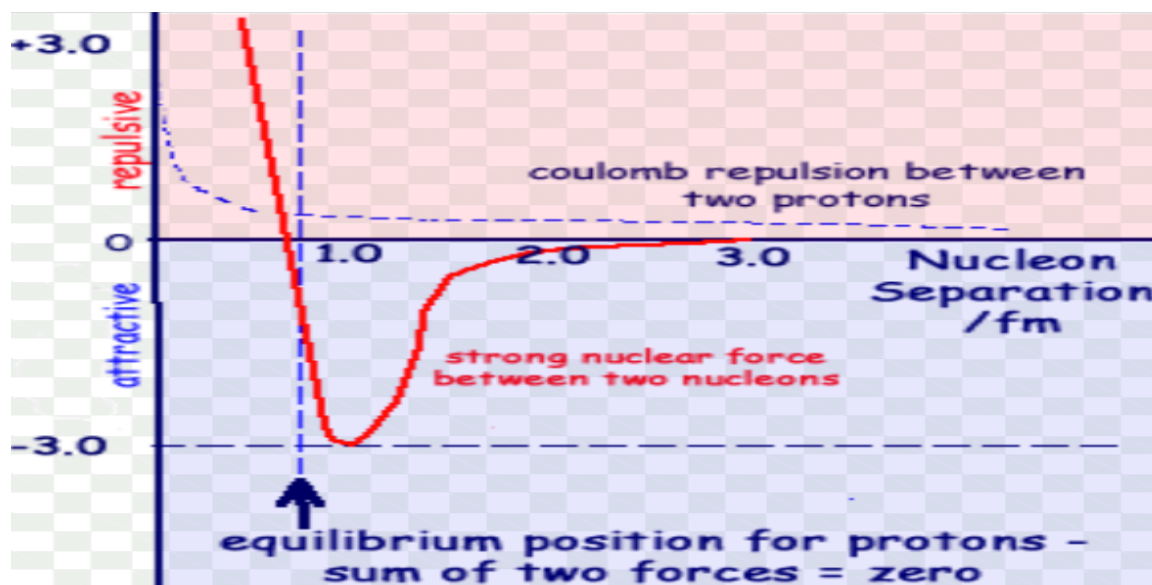
(2)(p-n force):-

The (p-n)force is examined by neutron-proton scattering experiment. At large distance of separation there is no force between two particles but at about 3fm neutron and proton attract one another by strong nuclear force.

(3)(n-n force):-

The neutron excess in heavy nuclei confirms that (n-n)forces are attractive but not sufficiently large to lead to a stable dineutron just as a stable diproton does not exist. The scattering length of (n-n) appears to have -ve sign.





At a nucleon separation of a femto metre - which is typical for nucleons within a nucleus - it is a very strong attractive force ( $10^4$  N).

The line for the strong nuclear force cuts the distance axis at about 0.5 fm and approaches it very closely at about 3.0 fm.

The equilibrium position - one with zero force acting - for nucleons in a nucleus is at a separation of about 0.5 fm (some sources say 0.4, 0.7, 0.8 fm).

At much smaller separations between nucleons, the force is very powerfully repulsive. In other words nuclei are not easily squashed as the nucleons won't move closer together!

A separation of greater than  $0.5 \times 10^{-15}$  m results in a smaller force of attraction - one that is virtually zero by about 3 femtometres. Thus, the strong nuclear force is a very short-range force.

### **Properties of nuclear forces:**

#### **1. Nuclear forces are short range forces:**

Nuclear forces are short range forces so each nucleon comes under the influence of only those nucleons which are in its close vicinity. Nuclear forces are appreciable only when the distance between nucleons is of order of 2.2 fm and vanishes at large distances at about 4.2 fm. If nuclear forces are long range forces then each neutron will come under influence of all other nucleons in nucleus. Then binding energy is proportional to:

- Number of nucleons exerting the force
- Number of nucleons on which force is being exerted

Hence binding energy is proportional to  $A^2$  but actually binding energy is proportional to  $A$ .

#### **2. Nuclear forces are charge independent:**

The three forces (p-p), (p-n) and (n-n) are equal in magnitude. Mirror nuclides are those nuclides which contain same number of nucleons but in which proton and neutron numbers are interchanged.

For example:

${}^1_1\text{H}^3$  contains 1 proton and 2 neutrons having binding energy 8.5 MeV and  ${}^2_2\text{He}^3$  contains 2 protons and 1 neutron having binding energy 7.7 MeV and are mirror nuclides. The three forces are equal in magnitudes, the difference of  $8.5 - 7.7 = 0.8$

MeV in binding energy is due to fact that (n-n)force in  ${}^1_1\text{H}^3$  is greater than (p-p)force in  ${}^2_2\text{He}^3$  due to electrostatic force of repulsion between protons.

### **3. Nuclear forces are strongest known forces in nature:**

The magnitude of nuclear forces is many times electrostatic repulsive force between protons and about  $10^{38}$  times the gravitational force between the neutrons.

### **4. Nuclear forces are spin dependent:**

The force of attraction between two nucleons having parallel spin is stronger than force between two nucleons having anti-parallel spin.

### **5. Nuclear forces have property of saturation:**

Each nucleon interacts with limited number of nucleons nearest to it. This factor is known as saturation of nuclear forces. Saturation depends on mass number A, if there is no saturation then each nucleon will interact with remaining (A-1) nucleons and binding energy is proportional to  $A(A-1)/2$  in other words it is proportional to  $A^2$ , but actually it is proportional to A. Hence nuclear forces are saturated forces.

### **6. Nuclear forces are Non-Central:**

The force existing between two nucleons has a non-central component that does not point along the line joining the two nucleon. This non central component depends upon how the nuclear spins are oriented relative to the line joining the nucleons.

### **7. Nuclear forces are exchange forces**

Nuclear forces have exchange character and they brought into existence due to exchange of  $\pi^0$ ,  $\pi^+$ , and  $\pi^-$  mesons between the nucleons.

### **8. Nuclear forces are Repulsive forces at too close distance**

At too close distance of approach between nucleons (at 0.5 fm or less), nuclear force become force of repulsion.

### **Meson theory of nuclear forces**

Nuclear forces have an exchange character as these forces are brought into existence due to exchange of pions between the nucleons.

It is supposed that inter nuclear forces in the nucleus are, at least in part ,some type of (exchange forces).

Energy distribution among beta particles emitted by radioactive nuclei, we assumed that a proton and neutron were two different state of same particle-the nucleon and beta particle was continuously being exchanged between them. But the force due to exchange of electrons was too small to account for observed nuclear energies.

### **Meson Theory:**

The meson theory started from the extension of the concept of the field of force so as to include the nuclear forces in addition to the gravitational and

electromagnetic forces. The necessity of introduction of specific nuclear forces, which could not be reduced to electromagnetic interactions between charged particles, was realized soon after the discovery of the neutron, which was to be bound strongly to the protons and other neutrons in the atomic nucleus. Specific nuclear forces between two nucleons, each of which can be either in Nobel Lecture, December 12, 1950 the neutron state or the proton state, must have a very short range of the order of  $10^{-13}$  cm, in order to account for the rapid increase of the binding energy from the deuteron to the alpha-particle. The binding energies of nuclei heavier than the alpha-particle do not increase as rapidly as if they were proportional to the square of the mass number A, i.e. the number of nucleons in each nucleus, but they are in fact approximately proportional to A. This indicates that nuclear forces are saturated for some reason.

The prediction of Yukawa was confirmed by experimental discovery of ( $\mu$ -meson) in 1936 and of ( $\pi$ -meson in 1947. binding nuclear energies were explained by pions. The pions may be neutral, positive and negative as  $\pi^0$ ,  $\pi^+$ , and  $\pi^-$ . The intrinsic spin of pion is zero.

#### **Mass of $\pi$ -mesons:**

Mass of  $\pi$ -mesons is estimated from Heisenberg's uncertainty principle. If  $\nabla E$  is rest mass energy of mesons, then this energy must be exchanged between the nucleons in time  $\nabla t$  so that;

$$\nabla E. \nabla t \approx h/2 \pi$$

The distance must not exceeds 1.4 fm. Velocity of light  $3 \times 10^8$  m/s

$$\nabla t = (1.4 \times 10^{-15}) / (3 \times 10^8)$$

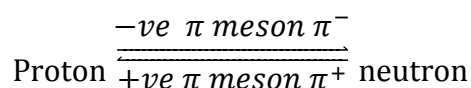
$$\nabla E = 140 \text{ MeV}$$

The mass of electron is = 0.51 MeV (approximately)

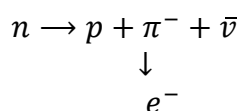
Mass of meson = 280 X mass of electron.

The experimental value comes out to be 270 times the mass of the electron which is in close agreement.

The attraction between a nuclear proton and neutron is represented as

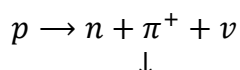


The emission of a  $\beta^-$  -particle by a nucleus and the consequent conversion of a neutron into a proton in the nucleus take place when a  $\pi^-$  meson is ejected. The  $\pi^-$  meson decays almost instantly and appears as a  $\beta^-$  - particle outside the nucleus.



Where  $n$  represents a neutron,  $p$  a proton,  $\bar{\nu}$  an anti-neutrino and  $e^-$  an electron.

The emission of a positron is represented as



$$e^+$$

Where  $\nu$  represent a neutrino and  $e^+$  a positron.

**Process of  $\pi$  – meson exchange:** To understand the process of  $\pi$  – meson exchange forces, the proton and the neutron are each supposed to be surrounded by a *virtual meson cloud*. The transfer of a  $\pi^+$  meson from the proton cloud to the neutron cloud only results in a change of identity of the particle *i.e.* in the final state we again have a proton and a neutron with the same energy as in the initial state. This is represented as under

$$p + n = n' + \pi^+ + n \rightarrow n' + p'$$

Where  $n'$  indicates the neutron formed from the proton and  $p'$  a proton formed from the neutron *i.e.* the “primes” indicate change in identity of the nucleons. This change in identity of the nucleons without change of energy can also take place through the following process.

$$n + p \rightarrow p' + \pi^- + p \rightarrow p' + n'$$

$$p + p \rightarrow p' + \pi^0 + p \rightarrow p' + p'$$

$$n + n \rightarrow n' + \pi^0 + n \rightarrow n' + n'$$

Thus, the force field between two protons or between two neutrons is carried by  $\pi^0$  meson. But the force field between a proton and neutron can be carried by a charged  $\pi^+$  or  $\pi^-$  meson. In this way, the charge is carried from one nucleon to other and proton is changed into a neutron and vice versa.

### **Facts in Support:**

The experimental values of magnetic moment of free proton and free neutron further support ‘Yukawa’s meson field theory. A free proton is, for a part of its life time a neutron with closely bound ( $\pi^+$ ) meson. Thus net magnetic moment of free proton will exceed that given by simple theory. Similarly a neutron is, for fraction of its life time dissociated into proton and ( $\pi^-$ ) meson. This will give negative magnetic moment.

The stable state implies force of attraction between the nucleons and vice versa. The actual situation is combination of two states.

**Radiation:** The propagation of energy from a radiative source to another medium is termed radiation. This transmission of energy can take the form of particulate radiation or electro- magnetic radiation (*i.e.*, electromagnetic waves). The various forms of radiation originating from atoms, which include (among others) visible light, X-rays and  $\gamma$ -rays, are grouped together under the terms “electromagnetic radiation” or “the electro- magnetic spectrum”. Radio waves, which have the longest wavelengths and thus the lowest frequencies and energies of the various types of electromagnetic radiation, are located at one end of the electromagnetic spectrum, whereas X-rays and  $\gamma$ -rays, which have the highest frequencies and energies, are situated at the other end of this spectrum.

## Photon:

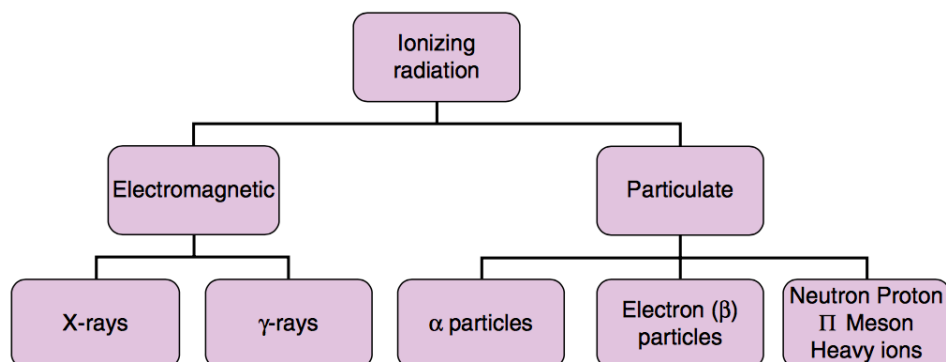
- If the smallest unit of an element is considered to be its atoms, the photon is the smallest unit of electromagnetic radiation.
- Photon has no mass.

Electromagnetic radiation can also be subdivided into ionizing and nonionizing radiations. Nonionizing radiations have wavelengths of  $\geq 10^{-7}$  m. Nonionizing radiations have energies of  $< 12$  electron volts (eV); **12 eV is considered to be the lowest energy that an ionizing radiation can possess.**

Types of nonionizing electromagnetic radiation:

- Radio waves
- Microwaves
- Infrared light
- Visible light
- Ultraviolet light

**Ionizing Radiation:** Ionizing (high-energy) radiation has the ability to remove electrons from atoms; i.e., to ionize the atoms. Ionizing radiation can be electromagnetic or particulate radiation. Clinical radiation oncology uses photons (electromagnetic) and electrons or (rarely) protons or neutrons (all three of which are particulate) as radiation in the treatment of malignancies and some benign conditions.



## The Interaction of Radiation with Matter

Radiation is scattered and absorbed when it passes through tissue. The intensities of monoenergetic X-rays or gamma rays attenuate exponentially

within tissues. In other words, the intensity of radiation constantly decreases as it propagates within tissues. This decrease depends on the type of tissue and its thickness. If the wavelength stays constant, the intensity of the radiation passing through a tissue can be calculated by Lambert's Law and is applicable for linear attenuation.

$$I = I_0 e^{-\mu(h\nu, Z)x}$$

Where  $I$  = intensity of outgoing radiation beam &  $I_0$  = intensity of incoming radiation beam

$\mu(h\nu, x)$  = absorption coefficient (which is positively correlated with the fourth power of the atomic number of the penetrated tissue, and the third power of the wavelength of the radiation)

$x$  = tissue thickness

### **THE WAYS IN WHICH PHOTONS MAY INTERACT WITH MATTER:**

Photoelectric Effect

Compton Effect (Incoherent Scattering)

Pair Production

Triplet Production

Raleigh and Thomson Scattering (Coherent Scattering)

Photodisintegration

#### **Photoelectric Effect (Fig. 1.19)**

This is the basic interaction in diagnostic radiology.

It is dominant at energies of less than 35 kV, and in atoms with high atomic numbers ( $Z$ ).

Since the atomic number of bone is higher than that of soft tissue, bone absorbs more radiation than soft tissue. This absorption difference is the basis of diagnostic radiology.

This effect also explains why metals with high atomic numbers (e.g., lead) are used to absorb low-energy X-rays and gamma rays.

The probability of the photoelectric effect occurring is inversely proportional to the cube of the photon energy ( $E^3$ ) and proportional to the cube of the atomic number ( $Z^3$ ).

### Compton Effect (Fig. 1.21)

This is the main mechanism for the absorption of ionizing radiation in radiotherapy.

It is the **dominant effect across a wide spectrum of energies, such as 35 kV–50 MV.**

It has no dependency on the atomic number ( $Z$ ) of the absorbent material, but it does **depend on the electron density of the material.**

The absorption of incoming radiation is the same for bone and soft tissues.

Importantly, incoherent scattering is not directly related to atomic number but rather the concentration of electrons in tissue. As atomic number rises, the density of electrons falls slowly, so incoherent scattering becomes slightly less likely in high  $Z$  materials. Incoherent scattering also decreases with energy.

### Pair Production

The threshold photon energy level for pair production is 1.02 MeV; below this, pair production will not occur.

The probability of pair production occurring increases as  $Z$  increases.

Pair production is more frequently observed than the Compton effect at energies of more than 10 MeV (Fig. 1.23).

- When pair production occurs in the field of an orbital electron, the effect is referred to as triplet production, and three particles (an electron–positron pair and the orbital electron) share the available energy. The threshold for this effect is  $4m_e c^2$ .

### 1.4.4

#### Coherent Effect (= Rayleigh Scattering, = Thomson Scattering)

Here, an electron is scattered when an electromagnetic wave or photon passes close to it [21]. This type of scattering is explained by the waveform of the electromagnetic radiation. There are two types of coherent scattering: Thomson scattering and Rayleigh scattering (Fig. 1.24). **The wave/photon only interacts with one electron in Thomson scattering**, while it interacts with all of the electrons of the atom in Rayleigh scattering. In Rayleigh scattering, low-energy radiation interacts with an electron, causing it to vibrate at its own frequency. Since the vibrating electron accelerates, the atom emits radiation and returns to its steady state. Thus, there is no overall transfer of energy to the atom in this event, so ionization does not occur. **The probability of coherent scattering is high in heavy (i.e., high- $Z$ ) matter and for low-energy photons.**

Coherent scattering is more probable as atomic number increases and as photon energy decreases. This makes it of some concern in diagnostic x-ray, where it can cause loss of contrast and blurring. For radiotherapy it has minimal impact on



attenuation.

**Table 1.1** Interaction probabilities for various photon energies in the photoelectric effect, the Compton effect, and pair production in water

Photon energy (MeV)	Interaction probability (%)		
	Photoelectric effect	Compton effect	Pair production
0.01	95	5	0
0.026	50	50	0
0.060	7	93	0
0.150	0	100	0
4.00	0	94	6
10.00	0	77	23
24.00	0	50	50
100.00	0	16	84

### **Photonuclear reactions :**

Photo nuclear reactions (also referred to as photodisintegration reactions) occur when a high energy photon is absorbed by the nucleus of an atom, resulting in an emission of a neutron ((x, n) reaction) or proton ((x, p) reaction) and a transformation of the nucleus into a radioactive reaction product.

- The threshold for a particular photonuclear reaction depends on the reaction and the nucleus and is of the order of 10 MeV or higher for most nuclei (with the exception of the deuteron and  ${}^9\text{Be}$  nuclei, for which the threshold is of the order of 2 MeV).
- The probability for photonuclear reactions is much smaller than that for other photon interaction, and their contribution to the total attenuation coefficient amounts to only a few percent at photon energies above the reaction threshold.
- While photonuclear reactions do not play an active role in photon attenuation consideration, they are of concern in high energy radiotherapy treatment rooms because of the neutron production through the (x, n) reactions and because of the radioactivity that is induced in the treatment room air and in machine components through the (x, n) reaction. Both the neutrons and the radioactivity pose a health hazard to personnel and must be dealt with in the treatment room and treatment machine design. The neutron problem is dealt with special treatment room doors incorporating borated hydrogenous materials to thermalize and absorb the neutron, the radioactivity with adequate room ventilation (six to eight air changes per hour) and use of machine components with a low reaction cross-section and short half-life of the reaction product.

### **Attenuation:**

When radiations pass through any material, a reduction in the beam intensity occurs. This is called attenuation. Attenuation is the reduction in amplitude and intensity of a signal.

### Half-Value Layer or Half-Value Thickness:

It is the thickness of the material at which the intensity of radiation entering it is reduced by one half. HVL can also be expressed in terms of air kerma rate (AKR), rather than intensity. Approximate half-value layers for a variety of materials against a source of gamma rays (Iridium-192):

Concrete: 44.5 mm, Steel: 12.7 mm, Lead: 4.8 mm, Tungsten: 3.3 mm, Uranium: 2.8 mm

### THE LINEAR ATTENUATION COEFFICIENT ( $\mu$ ):

The linear attenuation coefficient ( $\mu$ ) describes the fraction of a beam of x-rays or gamma rays that is absorbed or scattered per unit thickness of the absorber. This value basically accounts for the number of atoms in a cubic cm volume of material and the probability of a photon being scattered or absorbed from the nucleus or an electron of one of these atoms.

### Relation between Half Value Layer and Linear Attenuation Coefficient:

For a mono energetic beam of X-rays or gamma rays

$$I = I_0 e^{-\mu(h\nu, z)x}$$

If  $x = 1HVL$  then

$$I = 0.5 I_0 \text{ so}$$

$$0.5I_0 = I_0 e^{-\mu(h\nu, z)HVL}$$

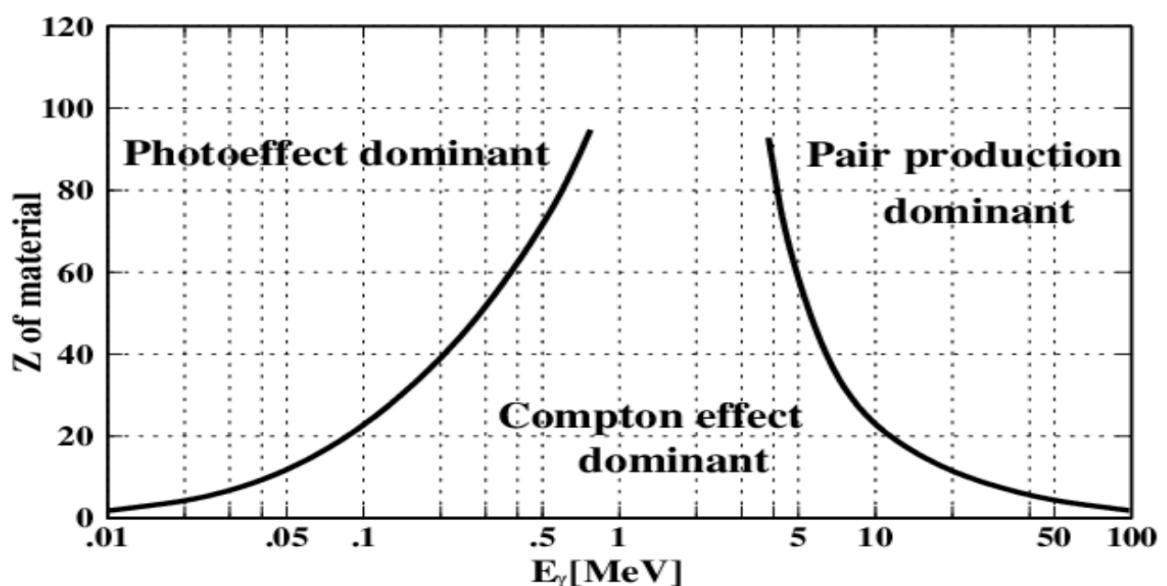
$$\ln 0.5 = \ln e^{-\mu(h\nu, z)HVL}$$

$$0.693 = \mu(h\nu, z)HVL$$

$$\frac{0.693}{\mu(h\nu, z)} = HVL$$

### Auger effect:

The Auger effect is a physical phenomenon in which the filling of an inner-shell vacancy of an atom is accompanied by the emission of an electron from the same atom. When a core electron is removed, leaving a vacancy, an electron from a higher energy level may fall into the vacancy, resulting in a release of energy. Although most often this energy is released in the form of an emitted photon, the energy can also be transferred to another electron, which is ejected from the atom; this second ejected electron is called an Auger electron.



### Properties of nuclear radiation used in detection instruments:

Detection instruments are used to detect sub-atomic particles or high energy photons. Sub-atomic particles are too small to be detected directly thus the necessity of suitable detecting instruments. The working of various detection instruments is based upon one of the following three main characteristic properties of the particles and raise emitted in nuclear interactions.

**1. Ionisation.** They ionize the gas through which they pass.

**2. Fluorescence.** They cause fluorescence in certain materials.

**3. Photographic effect.** They effect a photographic plate.

The following detecting devices depend upon ionization property.

(a) Ionization chamber

(b) Proportional counter

(c) Geiger Mueller counter

(d) Semi-conductor detector

(e) Wilson cloud chamber

(f) Bubble chamber.

The scintillation counter depends upon fluorescence. The nuclear emulsion detection depends upon photographic effect.

**(b) Ionization chamber.** The ionization chamber works on the principle that charged sub-atomic particles can ionize a gas. The number of ion pairs formed give information with regard to the nature of an incident particle as well as its energy.

### Ionization chamber are of two types:

**1. Pulse ionization chamber.** In this type of ionization chamber, the ionization produced by charged particles traversing the gas is detected as a single voltage pulse. This is known as non-integrating type.

**2. Current ionization chamber:** In this type of ionization chamber pulses are not separated but continuous accumulation of charge leads to flow of electric current. This is known as integrating type.

The working of two types depends upon the response time of the system relative to the frequency of arrival of the ionizing particles. **If the response time is small each pulse produced by the ionizing particle can be detected separately and counted. Thus the non integrated type must have a small response time** where as the integrating type must have a long response time.

**Construction:** An ionization chamber consists of a metallic cylinder C with a thin axial wire W enclosed in a glass envelop in which a suitable inert gas (like argon) is filled. A high potential difference is established between the cylinder and the wire electrode; the wire electrode being at a positive potential with respect to the cylinder. A resistance R, a capacitor and an amplifier cum pulse counter are connected as shown. The electric field is such that the loss of ions through recombination is kept as low as possible, at the same time ensuring that the ionization by collision does not occur. The ionization chamber thus operates under saturation condition.

**Working:** When a charged particle enters the active volume of the chamber, it ionizes the gas molecules and produces a large number of ion pairs along its path. The positive ion moves towards the cylindrical electrode and a negative ion (electrons) towards the positive wire electrode. The positive ions being heavier will move slowly.

Suppose the number of ion pairs produced =  $n$ . Then the height of the resulting voltage pulse  $\Delta V$  across the external resistor is given by

$\Delta V = \frac{ne}{C}$  where C is the capacitance of the capacitor formed by cylinder-wire system.

It has been calculated that 35eV of energy is required to form an ion pair in air. If the incoming particle has an energy of 1MeV and it loses all its energy in the chamber, then the number of ion pairs produced

$$n = \frac{1 \times 10^6}{35} = 2.86 \times 10^4. \text{ If the capacitance of the ionization chamber is } 10 \text{ pF} = 10 \times 10^{-12} \text{ F, then pulse height}$$

$$\Delta V = \frac{ne}{C} = \frac{2.86 \times 10^4 \times 1.6 \times 10^{-19}}{10 \times 10^{-12}} = 4.57 \times 10^{-4} \text{ V}$$

The voltage pulse or the source of e.m.f produces a flow of current i in the external resistance R is given by

$$I = \frac{ne}{CR}$$

If the time constant of the instrument (CR) is large as compared to the time between ionizing events a steady state is reached and a direct current may be measured. On the other hand if (CR) is small each pulse produced by an ionizing event may be recorded separately. If the individual particles are to be counted, then the pulses are fed to a pulse amplifier which is connected to the ionization chamber by a coupling capacitor. In the integrating type ionization chamber the total ionization current produced by the incident radiation is measured.

It has been found that if a grid is introduced in between the electrodes of an ionization chamber and maintained at half the potential of the electrodes, the response becomes very quick and performance greatly improved.

**Uses:** 1. Ionization chambers have been used to study  $\alpha$  - particles, protons and nuclei of lighter elements. These particles have large specific ionizations, produce strong pulses and are easily counted by the ionization chamber.

2. For neutron detection the chamber is filled with Boron trifluoride (where boron is enriched with  $^{10}_5\text{B}$ )

3. The ionization chambers are not used for electrons because of their low primary ionization. As  $\beta$  rays and  $\gamma$  rays do not produce enough ionization to give pulses detectable above the background noise, these cannot be detected by ionization chamber.

### **Linear Accelerator:**

For the study of nuclear reactions charged particles having energies of many million electron volts are required. It is difficult to generate direct voltages of the order of 10 million volts chiefly due to insulation difficulties. To obtain linear acceleration of a charged particle in excess of 10 MeV indirect methods are used. One of these is a linear accelerator.

In a linear accelerator a moderate accelerating potential is applied a number of times so that charged particles are accelerated along a straight line. A simple form of the linear accelerator is shown in fig 6.4. The charged particles or ions travel through an aperture A and move along the axis of a series of coaxial cylindrical electrodes 1, 2, 3, 4 etc. . . . These cylindrical electrodes are known as drift tubes. The drift tubes are connected to an A.C source of very high frequency say a high frequency oscillator so that alternate tubes have potentials of opposite signs. Thus in one half cycle if tubes 1 and 3 are positive, 2 and 4 will be negative.

After half a cycle the polarities are reversed i.e., 1 and 3 will be negative and 2

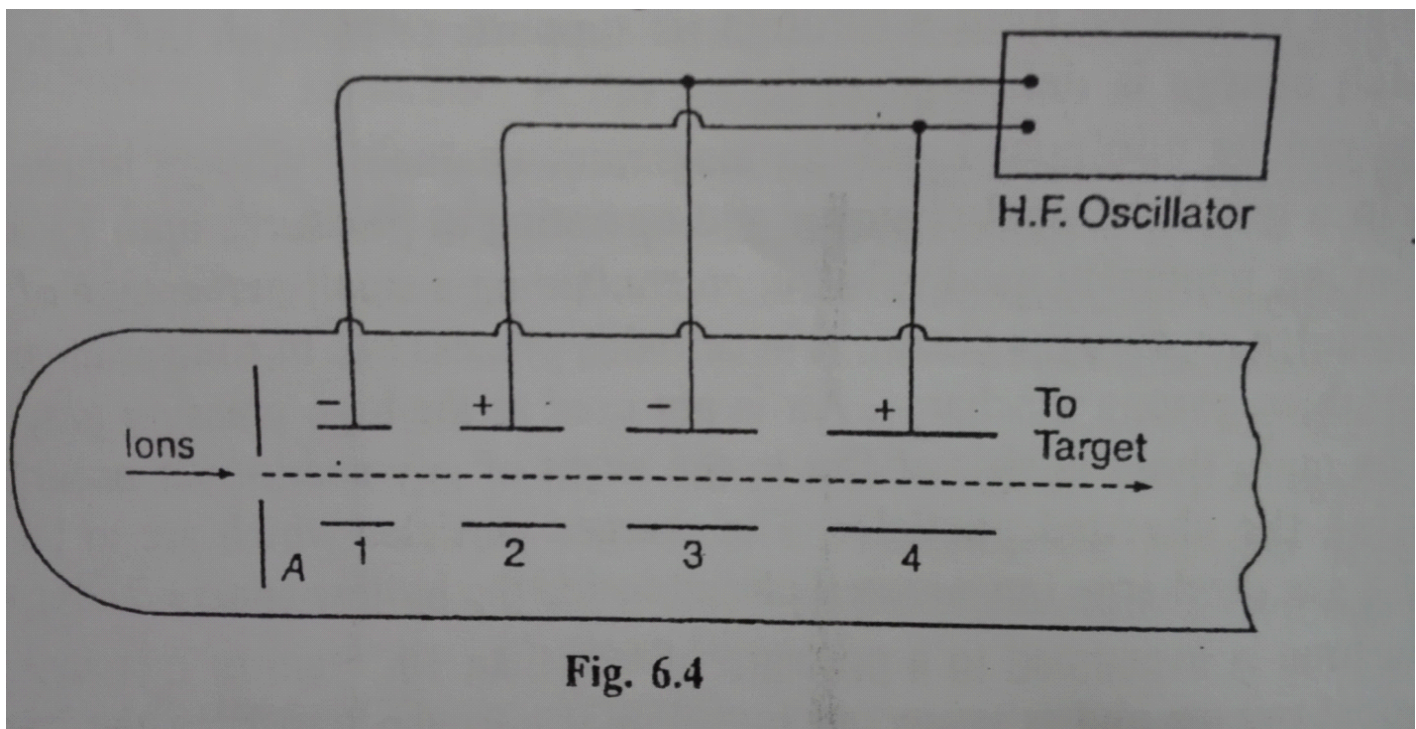


Fig. 6.4

and 4 positive.

Suppose a positive ion leaves A and is accelerated during the Half cycle when the drift tube No.

1 is negative with respect to A, then velocity  $v_1$  of the ion on reaching the drift tube is given by

$$\left(\frac{1}{2}\right)mv_1^2 = Ve$$

Or

$$V_1 = \sqrt{2Ve}/m$$

Where  $e$  is the charge and  $m$  the mass of the ion. It is supposed that  $v_1$  is small as compared to  $c$  the velocity of light so that the change in mass due to relativity effect is negligible. The ions are accelerated in the gap between the tubes but travel with constant velocity in the field free space within the tube themselves. The length of tube 1 is so adjusted that as the positive ions come out of it, the tube has positive potential and next tube No. 2 has negative potential, i.e., the potential change sign. The positive ion is again accelerated in the space between the tubes 1 and 2 and on reaching the tube 2 its velocity  $v_2$  is given by

$$\left(\frac{1}{2}\right)mv_2^2 = 2Ve$$

Or

$$V_2 = \sqrt{2} = \sqrt{2Ve}/m = \sqrt{2}v_1$$

This shows that  $v_2$  is  $\sqrt{2}$  times  $v_1$ . In order that this ion on coming out of tube 2 may find tube 3 just negative and the tube 2 positive, it must take the same time to travel through the tube 2. As its velocity is  $\sqrt{2}v_1$  the length of tube 2 must be

$\sqrt{2}$  times the length of tube 1. For successive accelerations in successive gaps the tubes 1,2,3,4 etc., must have lengths proportional to  $1, \sqrt{2}, \sqrt{3}, \sqrt{4}$  etc. to a first approximation.

Energy of the ion.

If  $n$  is the number of gaps that the ions travel in the accelerator and  $v_n$  is the final velocity acquired by it, then

Velocity of the ion as it emerges out of the  $n$ th tube

$$V_n = \sqrt{n} \sqrt{2eV/m}$$

Kinetic energy of the ion  $(1/2) m v_n^2 = nVe$

The final energy of the ions when they strike the target depends upon the overall length of the accelerator i.e, the total number of gaps and on the energy gained in each gap. The beam striking the target consists of pulses of particles. The number of pulses per second is equal to the frequency of the alternating voltage applied to the drift tubes.

### ***Length of the cylinder.***

As the ion is accelerated in the gap between two cylinder electrodes, the time taken by the ion to travel through the cylinder should be equal to half the time period of the high frequency voltage so that each time the ion comes out of the cylinder the polarity changes.

If  $v_n$  is the velocity of the ion, the time of passage through the  $n$ th cylinder of length  $L_n$

$$\begin{aligned} t &= L_n / V_n \\ &= T/2 \\ &= 1/2 * f \end{aligned}$$

Where  $f$  is the frequency of the oscillating electric field.

$$L_n = V_n / 2 * f = \sqrt{(2nVe/m)} / 2 * f$$

This equations shows that the length of the successive cylinder has to be increased in order to get a resonance acceleration of the ion at each gap and the length  $L_n \propto n^{1/2}$

The limitations of this accelerator are;

- The length of the accelerator becomes inconveniently large and it is difficult to maintain vacuum in such a large chamber.
- The current is available in the form of pulses of short duration.

**Q.6.4 (a) Describe the principle construction and working of a cyclotron .Drive expression for the maximum kinetic energy achieved by a particle of mass  $m$  in term of the applied magnetic field and Dee radius. Also state the relation in terms of the frequency of the applied electric field. Discuss its limitations.**

**(b) Can we accelerate neutrons by a cyclotron?**

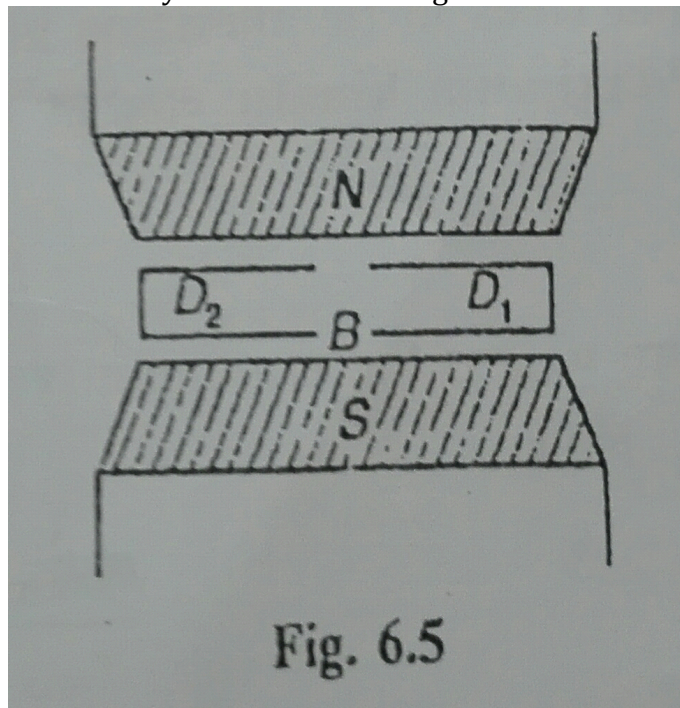
**ANS (a)**



## CYCLOTRON:

***The alpha and beta particles given out by natural radioactive substances neither possess sufficiently large speeds nor are their speeds under control.***

It was, therefore, felt necessary to accelerate charged particles to very high velocities by the application of electric and magnetic fields. Cockroft and Walton produced fast moving protons by electronic voltage multiplication device. The best arrangement was, however, made by Lawrence and Livingston in 1934 and is called a cyclotron. This arrangement won the Nobel prize for Lawrence.



### CONSTRUCTION:

The cyclotron consists of two D-shaped hollow metal segments D<sub>1</sub> and D<sub>2</sub> (called the Dees) lying in a horizontal plane with a small gap separating them. A magnetic field NS is applied perpendicular to the plane of the paper and D<sub>1</sub> and D<sub>2</sub> are connected to a high voltage, high frequency alternating current as shown in fig. 6.5.

### PRINCIPLE:

The cyclotron is a magnetic resonance type positive ion accelerator. The charged particle to be accelerated, rapidly passes through an

alternating electric field along a closed path, its energy being increased each time. A strong magnetic field is used to control the motion of the particles and to return them periodically to the region of the accelerating electric field. The particle passes definite points of the alternating electric field almost exactly when the field is in the same phase i.e. in resonance.

### THEORY and WORKING:

If a positive ion is generated at a point B, as shown in fig. 6.6, within the gap at a time when D<sub>1</sub> is at a positive potential and D<sub>2</sub> at a negative potential, it will be accelerated across the gap to D<sub>2</sub> and then

$$Ve = \frac{1}{2}mv^2_{\max}$$

Where V is the applied voltage and e and m are charge and mass of the particle respectively. When it is inside the conductor, it will not be acted upon by the electric field, but under the influence of the applied magnetic field having a flux density B, it will travel along a circular path, the radius r of which is given by

$$Mv^2/r = Bev$$

OR

$$r = mv/be$$

And finally emerges at C in the direction indicated. The time taken by the positive ion to travel the semi-circular path.

$$t = \pi/\omega$$

$$= \pi r / Y$$

$$= \pi m / Be$$

Where  $\omega$  is the angular velocity of the ion the circular path and

$$\omega = Be/m$$

The value of  $t$  is a constant being independent of velocity of the ion and the radius in which it travels. If the frequency of the applied voltage's adjusted in such a manner that it is reversed as soon as the particle comes out of D2, the particle at C will be accelerated across the gap to D1 and will describe a further circular path in D1. The radius of this semi-circle as well as speed of the particle will, now, be greater than that in the first case, but as proved above, the time taken by the particle to travel the semi-circle path in D1 will be the same. Every time the particle emerges out of the Dees, the direction of the voltage is reversed and the particle is of the Dees in the direction indicated, through the window W.

### MAXIMUM KINETIC ENERGY OF PARTICLE:

The final energy  $E$  of the charged particle is given by

$$E = \frac{1}{2} m v_{max}^2$$

Where  $v_{max}$  is the maximum velocity gained by the charged particle in its final orbit of radius  $r_{max}$ . Now

$$\frac{m v_{max}^2}{r_{max}} = Be v_{max}$$

$$v_{max} = \frac{Be r_{max}}{m}$$

$$E = (1/2) m v_{max}^2 = (1/2) (m B^2 e^2 / m^2) r_{max}^2$$

$$= (1/2) (B^2 e^2 r_{max}^2 / m)$$

This relation gives the maximum kinetic energy of the charged particle in terms of applied magnetic field and Dee radius.

The condition for optimal acceleration of the ion in the inter Dee graph is that the time taken by the ion to travel the semi-circular path ( $t$ ) is equal to half the time period ( $T$ ) of oscillation

Of the applied high frequency electric field i.e.

$$t = \frac{T}{2}$$

$$t = T/2$$

$$\text{Or } \pi m / Be = T/2$$

$$\text{Or } T = 2\pi m / Be$$

if  $f$  is the frequency of the oscillation electric field, then

$$f = \frac{1}{T} = \frac{Be}{2\pi m}$$

This is the basic cyclotron resonance equation.

Hence in terms of  $f$  the maximum energy of the charged particle

$$E = (1/2) (B^2 e^2 r_{max}^2 / m)$$

$$= (1/2) 4\pi^2 m (B^2 e^2 / 4\pi^2 m^2) r_{max}^2$$

$$= 2\pi^2 m f^2 r_{max}^2$$

The particles are ejected out of the cyclotron as pulse streams and not continuously.



With a comparatively small potential difference of the order of 50,000 volts, very fast moving particles can be produced. For example, if the particle makes 200 revolutions before emerging out it will gain a velocity equivalent to a total fall through a potential of

$$2 \times 5000 \times 200 = 20 \text{ million-volts}$$

If heavy hydrogen is used instead of ordinary hydrogen, a beam of high energy deuterons is obtained. As their mass is double they possess greater energy and are more useful as atomic projectiles.

Limitations of the cyclotron: The energy to which a particle can be accelerated in a cyclotron is limited due to change in mass with velocity. The mass of a particle, when moving with a velocity  $v$  is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where  $m_0$  is the rest mass and  $c$  the velocity of light. As already proved, the time taken by a particle to travel the semi-circular path is  $\pi m / Be = T/2$

$$\text{Frequency } f = \frac{1}{T} = \frac{Be}{2\pi m} = \frac{Be \sqrt{1 - \frac{v^2}{c^2}}}{2\pi m_0}$$

Hence the frequency of rotation of the charged particle decreases as the velocity increases. As a result it takes a longer time to complete its semi-circular path and the particle continuously goes on lagging behind the applied alternating potential difference till a stage is reached when it can no longer be accelerated further.

- **Field variation:** The frequency of the ion can be kept constant by taking  $B\sqrt{1 - v^2/c^2}$  a constant. For this purpose the value of the magnetic field  $B$  should increase as the velocity of the ion increases so that the product remains unchanged.
- **Frequency modulation:** In the alternative method, the frequency of the applied A.C. varies so that it is always equal to the frequency of the rotation of the ion. The machine in which the frequency of electric field is kept constant and magnetic field is varied is called Synchrotron, whereas a machine in which a magnetic field is kept constant and the frequency of the applied electric field is varied is known as a Frequency modulated cyclotron or Synchrony-cyclotron.

**(b)** A neutron cannot be accelerated by a cyclotron. A neutron carries no charge; it cannot be accelerated by the electric field between the two Dees. It can also be not acted upon by the magnetic field so that its path within the Dees cannot be regulated.

**Q. 6.5. (a) Show that the maximum radius of curvature of the path of a particle inside the dees of a cyclotron is proportional to the square root of the number of times it crosses the gap between the dees.**

**(b) What are the primary functions of (i) Electric field (ii) Magnetic field in a cyclotron.**

Ans. Maximum radius of curvature. If  $V$  is the average voltage applied between the dees of a cyclotron and charge particle crosses the gap between the dees  $n$  times to reach the orbit of maximum radii, then energy acquired by the ion having a charge  $e$  is given by

$$E_{\max} = neV$$

$$E_{\max} = \frac{1}{2}(B^2 e^2 r_{\max}^2 / m)$$

$$B^2 e^2 r_{\max}^2 / 2m = neV$$

$$r_{\max} = \frac{1}{B} \sqrt{(2mV/e)} \sqrt{n}$$

If the values of  $B$  and  $V$  are kept constant,  $r_{\max}$  is directly proportional to the square root of the number of times the particle crosses the gap between the dees.

(b) (i) Electric field. The primary function of the electric field is to provide a potential difference between the dees of the cyclotron to accelerate the charge particle. Thus, an alternating electric field having a frequency such that the time taken by the particle to travel through the semicircular path within the dees is equal to half the time period of the alternating electric field.

(ii) Magnetic field. The primary function of the magnetic field is to move the charge particle into a semicircular path within the dees.

***Q.6.6 (a) can a cyclotron be used to accelerate electron ? If not why ?***

***(b) The pole pieces of a cyclotron are 1.2m in diameter and provide a magnetic field of 1.6 Wbm<sup>-2</sup>. What will be the energy of alphas, deuterons and protons in such a machine ? what should be the range of oscillator frequency to cover the acceleration of the above particles.***

***Mass of proton = 1.6x10<sup>-27</sup> kg and charge 1.6x10<sup>-19</sup> C.***

**Ans.(a)** Acceleration of electrons: A cyclotron can not be used to produce high energy electron beam. The reason for the same is that there is an appreciable increase in the mass of electron at fairly low energies. For example a ten percent increase in the rest mass of electron takes place at an energy of 50 KeV only. Electron being a very light particle there is an appreciable increase in its velocity at low energies which is not the case with massive particles like the proton or the alpha particle.

(b) Magnetic field  $B = 1.6 \text{ Wbm}^{-2}$

Radius of pole pieces  $r(\max) = 0.6 \text{ m}$

Mass of the proton  $m = 1.6 \times 10^{-27} \text{ kg}$

Charge on the proton  $e = 1.6 \times 10^{-19} \text{ C}$

Energy of the proton

$$E = B^2 e^2 r^2 / 2m$$

$$= (1.6 \times 1.6 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 0.6 \times 0.6) / (1.67 \times 10^{-27})$$

$$= 7.065 \times 10^{-12} \text{ J}$$

$$= 7.065 \times 10^{-12} / 1.6 \times 10^{-13} = 44.16 \text{ MeV}$$

Energy of deuteron. A deuteron has the same charge as proton but nearly twice the mass.

Hence  $e^2/m$  for deuteron =  $1/2$  that of proton. The energy of deuteron is therefore  $1/2 E = 22.08 \text{ MeV}$

Energy of alpha particle. An alpha particle has twice the charge and nearly four times the mass as that of a proton. Hence  $e^2/m$  for a particle =  $2 \times 2 / 4 = 1$  that of the proton. The energy of the alpha particle is therefore  $E = 44.16 \text{ MeV}$ .

Oscillator frequency  $f = Be / 2\pi m$

Oscillator frequency for proton  
 $= 1.6 \times 1.6 \times 10^{-19} / 2\pi \times 1.67 \times 10^{-27}$   
 $= 24.40 \times 10^6$  cycles/sec  
 Oscillator frequency for deuteron.

$= 24.40 \times 10^6 / 2$   
 $= 12.20 \times 10^6$  cycles/sec  
 Oscillator frequency for alpha particles  
 $= 24.40 \times 10^6 / 2$   
 $= 12.20 \times 10^6$  cycles/sec

**Q.6.7. A cyclotron is so called resonance device explain .Does betatron also depends upon resonance ?**

**Ans.** Cyclotron-a resonance device. In a cyclotron the value of magnetic field strength upon the frequency of oscillating electric field applied b/w the dees. It is so chosen as to give resonance b/w of arrival of charged particle in the gap and the reversal of the voltage b/w the dees this is done by the time adjusting taken by the charged particles to describe a semi circular path equal to half the time period of oscillations of applied high frequency electric field i.e...

$$\pi/\omega = T/2$$

$$f = 1/T = \omega/2\pi = Be/2\pi m$$

This gives a cyclotron resonance condition for a charged particle of a given value  $e/m$ .

If this condition is not satisfied there will be no resonance b/w the arrival of charged particle in the gap and reversal of voltage b/w the dees the particle goes out and will not be accelerated. This is why it is said that a cyclotron is a resonance device. Betatron is not a resonance device. The action of a betatron is like that of a transformer in which an alternating current applied to a primary coil induces similar current in secondary windings.

**Q. 6.8. A cyclotron with dees of radius 90 cm has a transverse magnetic field of 0.8 Tesla.**

**Calculate the energies to which (1) A proton and (2) Deutrons are accelerated.**

**Given**

**Mass of the proton =  $1.67 \times 10^{-27}$  kg.**

**Mass of deuteron =  $3.34 \times 10^{-27}$  kg.**

**Ans. (1) Proton.** Mass of the proton  $m = 1.67 \times 10^{-27}$  kg  
 Charge on the proton  $e = 1.6 \times 10^{-19}$  kg

Energy of the emergent proton  $E = 1/2(B^2 e^2 r^2 / m)$

$$= 0.8 \times 0.8 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 0.9 \times 0.9 / 2 \times 1.67 \times 10^{-27}$$

$$= 0.3973 \times 10^{-11} \text{ J}$$

$$= 24.8 \text{ MeV.}$$

**(2) Deuteron.** Mass of deuteron  $m = 3.34 \times 10^{-27}$  kg

Charge on the deuteron  $e = 1.6 \times 10^{-19} \text{ C}$

$$\begin{aligned}\text{Energy of emergent deuteron} &= 0.8 \times 0.8 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 0.9 \times 0.9 / 2 \times 3.34 \times 10^{-27} \\ &= 0.1984 \times 10^{-11} \text{ J} \\ &= 12.4 \text{ MeV.}\end{aligned}$$

**Q. 6.9. Between the Dee's of a cyclotron 1.5 meter in diameter an alternating potential difference of 15 mega cycles is applied. Calculate the energy in MeV of the protons issuing out of the cyclotron. Mass of proton =  $1.672 \times 10^{-27} \text{ kg}$ .**

**Ans.** In terms of frequency of the applied and the radius of cyclotron Dee's, the energy is

Given by

$$E = 2m$$

$$\text{Mass of proton} = 1.672 \times 10^{-27} \text{ and } f = 15 \times 10^6$$

$$E \text{ (in MeV)} = 2\pi^2 \times 1.67 \times 10^{-27} \times 15 \times 10^6 \times 15 \times 10^6 \times 0.75 \times 0.75 / 1.6 \times 10^{-19} = 26.11 \text{ MeV.}$$

**Q. 6.10. A uniform magnetic field of 2 Web/ is used in a cyclotron to accelerate protons.**

**The radius of the cyclotron is 0.32 meter. Calculate (1) how rapidly the electric field between**

**The dees should be reversed and (2) the frequency of the accelerated proton. Mass of proton =  $1.67 \times 10^{-24} \text{ gm}$  and charge =  $1.60 \times 10^{-19} \text{ C}$ .**

**Ans.** Magnetic field  $B = 2 \text{ Web/}$   
Mass of proton  $m = 1.67 \times 10^{-27} \text{ kg}$   
Charge  $e = 1.6 \times 10^{-19} \text{ C}$   
And  $r = 0.32 \text{ m.}$

**(1) The electric field between the Dee's should be reversed as soon as the proton completes**

**its journey in the semi-circular path of radius r with the velocity v. The time t after which the electric field should be reversed is given by**

$$\begin{aligned}t &= \pi r / v \\ &= \pi m / Be \\ &= \pi \times 1.67 \times 10^{-27} / 2 \times 1.6 \times 10^{-19} \\ &= 1.640 \times 10^{-8} \text{ sec.}\end{aligned}$$

Hence the electric field between the Dees should be reversed after every  $1.640 \times 10^{-8} \text{ sec.}$

Hence the number of times the protons comes out of the gap

$$\begin{aligned}n &= 1/t \\ &= 10^8 / 1.640 \\ &= 60.97 \times 10^6\end{aligned}$$

The frequency of the accelerated proton is given by the number of complete revolutions made

By it per second.

The time taken by the proton to complete one revolution

$$T = 2\pi r/v$$

$$F = v/2\pi r$$

$$\therefore \text{Frequency} \quad f = 1/T = 30.49 \times 10^6 \text{ cycles/sec}$$

**Q. 6.11. A cyclotron with Dee's of diameter 1.8 m has a magnetic field of 0.8 tesla.**

**Calculate the energy to which the doubly ionized helium ion  $\text{He}^{++}$  can be accelerated. Also**

**Calculate the number of revolutions the particle makes in attaining this energy. Mass of  $\text{He}^{++} = 6.68 \times 10^{-27} \text{ kg}$ .**

**Ans.** Mass of the  $\alpha$ -particle ( $\text{He}^{++}$ )  $m = 6.68 \times 10^{-27} \text{ kg}$ .

Charge on  $\text{He}^{++}$  ion,  $e = 2 \times 1.6 \times 10^{-19} \text{ C}$

Now  $E = (B^2 e^2 r^2 / 2m)$

$$B = 0.8 \text{ Tesla}; \quad r = 0.9 \text{ m}$$

$$E = 0.8^2 \times 1.6^2 \times 10^{-38} \times 0.9^2 / 2 \times 6.68 \times 10^{-27}$$

$$= 0.397 \times 10^{-11} \text{ J}$$

$$= 24.8 \text{ MeV.}$$

The frequency of the alternating electric field is given by

$$f = Be/2\pi m$$

$$= 0.8^2 \times 1.6^2 \times 10^{-38} / 2\pi \times 6.68 \times 10^{-27}$$

$$= 0.061 \times 10^8 = 6.1 \times 10^6 \text{ s}^{-1}$$

This gives the number of times the  $\text{He}^{++}$  ion comes out of the gap between the Dees each time

Undertaking a Semi-circular path.

$\therefore$  Number of complete revolution made by  $\text{He}^{++}$  ion in attaining the above energy

$$= f/2 = 1/26.1 \times 10^6 = 3.05 \times 10^6 \text{ s}^{-1}$$

**Q. 6.12. Deuterons in a cyclotron describe a circle of a radius 0.32m just before emerging**

**Out of the Dee's. The frequency of the applied e.m.f. is 10 MHz. Find the flux density of**

**The magnetic field and the velocity of the deuterons emerging out of the cyclotron. Mass of**

**Deuteron is  $3.32 \times 10^{-17} \text{ kg}$  and charge  $1.6 \times 10^{-19} \text{ C}$**

**Ans.** The frequency of the applied electric field is given by

$$f = Be/2\pi m$$

$$\therefore B = 2\pi mf/e$$

Here

$$m = 3.32 \times 10^{-17} \text{ kg};$$

$$f = 10 \text{ MHz} = 10 \times 10^6 \text{ Hz};$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$B = 1.304 \text{ Tesla}$$

Radius of the circle just before deuterons emerge  $r_{\text{max}} = 0.32 \text{ m}$

Now  $Bev = mv^2/r_{\text{max}}$

$$v = Ber_{\text{max}}/m$$

$$= 1.304 \times 1.6 \times 10^{-19} \times 0.32$$

$$v = 2.01 \times 10^7 \text{ ms}^{-1}$$

**Q. 6.13. What is a betatron? Derive the betatron condition for successful acceleration of electrons. Briefly describe its principle, construction and function of alternating magnetic field in it. (P.U. 1997, 1996, 1995, 1993; G.N.D. U. 1997, 1996; H.P.U. 1995, 1993; Cal. U. 1992)**

**Ans. Betatron.** The betatron is used to accelerate electrons to high energies. Since  $\beta$ -particles are fast moving electrons the accelerator is known as Betatron. It provides electrons which are much more energetic than  $\beta$ -particles emitted by natural radioactive nuclides. With a betatron it has been possible to obtain electrons at **300MeV** of energy. *Cyclotrons and synchrocyclotrons cannot be used to produce high energy electron beams because of the appreciable mass increase of the electrons at fairly low energies.* For example, a 10% increase in the rest mass of the electron takes place at an energy of 50 KeV only.

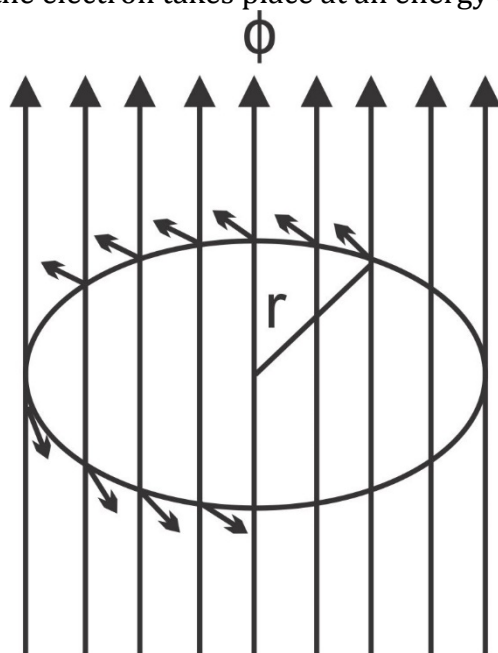


Fig. 6.7

**Betatron condition:** In a betatron, the electrons are accelerated with the help of an electric field produced by changing magnetic field. The electrons are maintained in a circular orbit by a magnetic field and at the same time these electrons are given energy by an induced emf resulting from an increase in this magnetic field. The principle can be explained by considering Fig.6.7 which represents an electron moving in an orbit of radius  $r$  where the total magnetic flux through the orbit is  $\phi$  and the flux density (magnetic induction) at the orbit itself is  $B$ , the lines of magnetic induction being perpendicular to the plane of orbit. If the total magnetic flux  $\phi$  is linked with the orbit is increased at the rate of  $\frac{d\phi}{dt}$ ,

then the induced e.m.f. in the electron orbit

$$E = -\frac{d\phi}{dt}$$

The work done on the electron in one revolution

$$= Ee$$

Where  $e$  is the charge on the electron. Now the force acting on the electron acts along the tangent to the circular path at any point and if  $F$  is this force, then

Work done by the electron in one revolution

$$= F \cdot 2\pi r$$

$\therefore$

$$Ee = F \cdot 2\pi r$$

Or

$$-e \frac{d\phi}{dt} = F \cdot 2\pi r$$

Or

$$F = -\frac{e}{2\pi r} \cdot \frac{d\phi}{dt}$$

As the energy of the electron increases, it will try to move in a larger orbit as the applied force increases. If a stable orbit of constant radius is maintained, this tendency of the electron is resisted.

As the flux density of the orbit is  $B$ , the force acting on the electron =  $Bev$  where  $v$  is the velocity of the electron. This force acts inward along the radius. If  $m$  is the mass of electron, then

$$\text{Centripetal force } \frac{mv^2}{r} = Bev$$

$$\therefore \text{Momentum of the electron} = mv = Ber$$

Hence the tangential force on the electron is equal to the rate of change of momentum

$$= er \frac{dB}{dt}$$

If a stable orbit of constant radius is to be maintained, the net tangential force on the electron must be zero.

$$\therefore -\frac{e}{2\pi r} \cdot \frac{d\phi}{dt} + er \frac{dB}{dt} = 0$$

$$\text{Or } \frac{d\phi}{dt} = 2\pi r^2 \frac{dB}{dt}$$

$$\therefore d\phi = 2\pi r^2 dB$$

Integrating both sides between the limits 0 and  $\phi$  and  $B$ , we have

$$\phi = 2\pi r^2 B$$

$$B = \frac{\phi}{2\pi r^2}$$

If  $B'$  is the average magnetic flux density over the whole area of the orbit, then

$$B' = \frac{\phi}{\pi r^2}$$

$$\therefore B = \frac{1}{2} B'$$

Thus, for the electron to be accelerated in a circular orbit of constant radius, the magnetic field should be non-uniform in such a way that at any instant the magnetic field at the orbit is just one half the average magnetic field (or flux) through the orbit. Equation (iii) represents condition for successful working for betatron to produce acceleration in electrons and is called betatron condition. The desired field distribution is obtained by using pole pieces of special shape so that the field at the Centre of the orbit is greater than that at the circumference of the orbit.

### Principle:

The principle underlying the working of the betatron is the betatron condition which states 'For electrons to be accelerated in a circular orbit of constant radius the magnetic field should be non-uniform in such a way that at any instant the magnetic field at the periphery of the orbit is just one half the average magnetic field through the orbit.'

**Construction and working:** The construction of the betatron is shown in fig. 6.8. The electrons are accelerated in highly evacuated tube DD called the dough-nut (pronounced do-nut). This tube is made of glass in small betatrons and of ceramic in large betatrons. It is placed between specially shaped pole pieces of an electromagnet energized by passing alternating current from 50 cycle mains supply through a pair of coils  $P_1P_1$  and  $P_2P_2$ . An increasing magnetic flux in a given direction is only obtained during the quarter cycle in which current increases from zero to its maximum value. The electrons are thus accelerated for a time  $\frac{1}{200}$  second at intervals of  $\frac{1}{50}$  sec.

To introduce the electrons into the stable orbit, an electron gun is used. It consists of a filament  $F$  which gives out thermionic electrons, a focusing grid  $G$

and the positive plate P. the gun is given a potential difference from an injector unit so that the electrons are introduced during the quarter cycle the magnetic flux is increasing. The electrons injected in a plate close to the stable orbit position will be accelerated by the magnetic field and execute a damped oscillatory motion in the beginning but will finally settle in the orbit. During  $\frac{1}{200}$  sec.

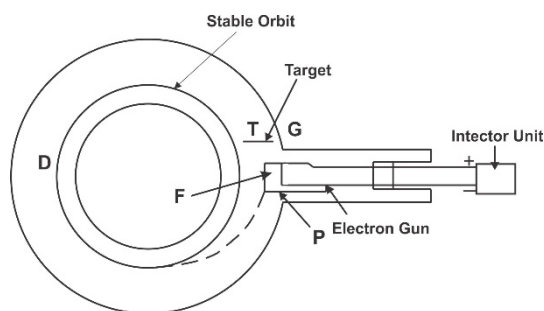


Fig. 6.9

When the magnetic field is increasing the electron will make several hundred thousand revolutions in the stable orbit gaining energy continuously. The electrons must be ejected from the betatron when the magnetic field reaches its maximum value, otherwise the electrons would slow down as the magnetic flux decreases and finally will reverse direction. The electrons are deflected from their stable orbit by sending a pulse of current through an auxiliary coil. The high energy electron beam can either be made to strike a target T within the tube, thus producing an intense X-ray beam (more correctly  $\gamma$ -rays) or electron beam can be removed through a window. The energy of the  $\gamma$ -rays obtained from a betatron is very much greater than that possessed by natural  $\gamma$ -rays, the order being about 300 MeV. These  $\gamma$ -rays are used for research in nuclear physics, deep X-ray therapy and for examination of metallurgical specimens.

Main functions of alternating magnetic fields. The main functions of alternating magnetic fields are:

- (i) The alternating or changing magnetic field gives rise to an electric field. The increasing magnetic field produces an e.m.f. which imparts energy to the electrons.
- (ii) With the help of the magnetic field the electron is maintained in the circular orbit.

**Q. 6.14. (a) Calculate the average energy gained per turn and the final energy of an electron in a betatron.**

**(b) A betatron has the following parameters**

Maximum magnetic flux at the orbit = 4000 gauss

Operating frequency = 60 Hz

Stable orbit diameter = 1.68 m

Find the average energy per turn and also final energy.

(a) Energy of electron. Let us suppose the magnetic flux in betatron is given by the relation

$$\Phi = \Phi_0 \sin \omega t$$

As an increasing magnetic flux in a given direction is only obtained during the quarter cycle in which the current in the electromagnet increases from zero to maximum value.

$$\text{Time of acceleration} = \frac{T}{4} = \frac{1}{4} \frac{2\pi}{\omega} = \frac{\pi}{2\omega}$$

Where T is the time period of the changing magnetic flux and  $\omega$  is the corresponding angular frequency.



energy gained by electron per cycle =  $Ee$

$$= e \frac{d\phi}{dt} = e \frac{d}{dt} \phi_0 \sin \omega t = e \phi_0 \frac{d}{dt} (\sin \omega t)$$

As this energy is gained in time  $\frac{T}{4} = \frac{\pi}{2\omega}$

Average value of energy per turn

$$= \frac{e\phi_0}{\frac{\pi}{2\omega}} \int_0^{\pi/2} \frac{d}{dt} (\sin \omega t) dt$$

$$= \frac{2e\omega\phi_0}{\pi}$$

Substituting the value of  $\phi_0 = 2\pi r^2 B$  From eq (ii) Q. No.13 we get

$$\text{average energy per turn} = \frac{2e\omega}{\pi} 2\pi r^2 B = 4e\omega r^2 B$$

For most of the time the electrons travel with a velocity equal

To the velocity of light =  $c$

$$\text{Total distance travelled in the process} = c \frac{T}{4} = \frac{c\pi}{2\omega}$$

If  $r$  is the radius of the orbit then

Number of revolution made by electron

$$N = \frac{c\pi}{2\omega \times 2\pi r} = \frac{c}{4\omega r}$$

Total (final) energy = Number of revolution made \* Average energy per revolution

$$= \frac{c}{4\omega r} \times 4e\omega r^2 B = cerB$$

The final energy of electron can also be estimated by using the relativistic relation for energy.

$$\text{Energy} = pc = cerB$$

As

$$p = Ber \quad [\text{see equation (1) Q.No.6.13}]$$

(b) Maximum magnetic flux at  $B=0.4T$

Stable orbit radius  $r=0.84 \text{ m}$

Operating frequency  $=60 \text{ Hz}$  As  $\omega = 2\pi 60$

Average Energy per turn  $= 4e\omega r^2 B$

$$= 4 \times 1.6 \times 10^{-19} \times 2\pi \times 60 \times 0.84 \times 0.84 \times 0.4 \text{ Joule}$$

$$= \frac{4 \times 1.6 \times 10^{-19} \times 2\pi \times 60 \times 0.84 \times 0.84 \times 0.4}{1.6 \times 10^{-19}} \text{ eV}$$

$$425.7 \text{ eV}$$

$$\text{Finel energy} = cerB$$

$$= 3 \times 10^8 \times 1.6 \times 10^{-19} \times 0.84 \times 0.4 \text{ Joule}$$

$$\frac{3 \times 10^8 \times 1.6 \times 10^{-19} \times 0.84 \times 0.4}{1.6 \times 10^{-13}}$$

$$= 100.8 \text{ MeV}$$

**Q.6.15: A betatron working on an operating frequency of 60 Hertz has stable orbit of diameter 1.6 m. find the energy gained per turn as also final energy if the magnetic field at the orbit is 0.5 Tesla .**

Ans Magnetic field at the orbit  $B= 0.5 \text{ T}$

$$\text{Stable orbit radius} = \frac{1.6}{2} = 0.8 \text{ m}$$

$$\text{operating frequency} = 60 \text{ Hz}$$

As

$$\omega = 2\pi \times 60$$

$$\text{Average energy per turn} = 4e\omega r^2 B$$

$$= \frac{4 \times 1.6 \times 10^{-19} \times 2\pi \times 60 \times 0.8 \times 0.8 \times 0.5}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 482.6 \text{ eV}$$

$$\text{Final energy} = ecrB = \frac{3 \times 10^8 \times 1.6 \times 10^{-19} \times 0.8 \times 0.5}{1.6 \times 10^{-13}} = 120 \text{ M}$$

**Q.6.16 (a) Explain the concept that the working of a betatron is like that of a transformer**

**(P.U.1991)**

**(b) A Betatron of 100 MeV energy has a stable radius of 0.84 m. Calculate ( i ) the value of magnetic field intensity at the orbit for this energy and ( ii ) the frequency of the applied electric field if average energy gain per turn = 420 eV**

**Ans. (a) Betatron-like a transformer.** The action of the betatron depends upon the same principle as that of the transformer in which an alternating current applied to a primary coil induces a similar current in the secondary windings. The primary current produces an oscillating magnetic field which in turn induces an oscillating e.m.f. in the secondary coil. The betatron is also like a transformer in which a cloud of electron located inside a dough-nut shaped vacuum chamber takes the place of the secondary winding. The chamber is placed within the pole pieces of an electro-magnet energized by an alternating pulsed current and the magnet produces a strong varying field in the central place or hole of the dough-nut. The electrons move in a circular orbit of constant radius within the vacuum chamber and gain energy by induction because of the change with time of the magnetic flux linking the orbit. Thus the electro-magnet plays the role of the primary coil.

**(b)** The final energy of the electron = 100 MeV

$$= 100 \times 1.6 \times 10^{-13} \text{ J}$$

Now final energy  $E = ecrB$

$$(i) \therefore \text{Magnetic field } B = \frac{E}{ecr} = \frac{100 \times 1.6 \times 10^{-13}}{3 \times 10^8 \times 10^{-19} \times 0.84}$$

$$= 0.397 \text{ T (say 0.4 T)}$$

Average energy per turn =  $4e\omega r^2 B$

If  $f$  is the frequency of the applied field, then it is =  $4e2\pi fr^2 B$

$$(ii) \therefore \text{Frequency } f = \frac{\text{Average energy per turn}}{8\pi r^2 B}$$

$$= \frac{420 \times 1.6 \times 10^{-19}}{8\pi \times 1.6 \times 10^{-19} \times 0.84 \times 0.84 \times 0.397} = 59.65 \text{ (say 60 cycles/sec)}$$

**Q. No. 1: Choose the best answer/Mark True or False/Fill in the blanks.**

- As atomic number rises, the density of electrons .....  
a) increases                      b) decreases                      c) is constant                      d) decreases fastly
- Coherent scattering is more probable as atomic number .....  
a) increases                      b) decreases                      c) exponentially decreases  
d) constant
- The probability of pair production and Compton effect is same at .....  
a) 0.026 keV                      b) 0.026 MeV                      c) 24 keV  
d) 24 MeV
- Binding energy of helium is .....  
a)  $45 \times 10^{-12} \mu\text{J}$                       b)  $4.5 \times 10^{-12} \mu\text{J}$                       c) 28J  
d) 4.5 J
- The energy equivalent of carbon is .....  
a) 11.2 ev                      b) 11.2 KeV                      c) 11.2 MeV                      d) 11.2 GeV
- The packing fraction is positive for elements having mass number below .....
- The nucleus is unstable if the neutron-proton ratio is less than 1:1 or greater than .....
- As the nucleus gets bigger, the electrostatic repulsions between the protons get .....
- Quantum chromodynamics (QCD) is the theory of the weak interaction between quarks and gluons. (True/False)
- The intrinsic spin of pion is zero. (True/False)

BS Sem-VII/MSc Sem-III		Nuclear Phys-I(Phy-01701/21301)	
Mid Term Exam.		Max. Marks: 30	
		Time Allowed: 1hr 05 min	
<b>Q#2</b>	Solve the Q#2 on the space below the questions.		
I	What are the applications of nuclear physics?		2
II	What are the limitations of Thomson plum pudding Model?		2
III	What are the indicators of nuclear stability?		2
IV	Write the bulk properties of binding energy.		2
V	Name the properties of nuclear forces.		2
<b>Q#3</b>			
I	Calculate the binding energy of ${}_{52}\text{Te}^{126}$ having mass 125.903322 amu?		2
II	Suppose now that you want to use aluminum to shield the X-rays of 30KeV. What thickness of aluminum is needed to the reduce the intensity by a factor of 1000? ( $\mu = 1.12 \text{ cm}^2/\text{g}$ , The density of aluminum is $2.7 \text{ g/cm}^3$ )		3
III	Find the mass of $\pi$ -meson. Write the conversion of neutron to proton & proton into neutron.		5

**BS Sem-VII/MSc Sem-III      Nuclear Phys-I(Phy-01701/21301)**  
**Final Term Exam Max. Marks: 20      Time Allowed: 20 min**

Q. No. 1: Choose the best answer/Mark True or False/Fill in the blanks.

- Which of these is associated with Alpha Radiation?  
 a)  $4/2$                       b)  $0/0$                       c)  $0/-1$                       d)  $0/1$
- Which type of radiation is the least penetrating?  
 a) alpha                      b) beta                      c) gamma                      d) x-ray                      e) neutron
- Betatron works on the principle of  
 a) Superposition    b) Ampere's law                      c) electromagnetic induction  
 d) self-induction
- In betatron, the electron moves in orbit of  
 a) constant radius                      b) variables radius    c) radius as  $(r^2)$                       d) radius as  $1/r^2$
- In betatron, the electrons are injected during.....magnetic field.  
 a) decreasing                      b) peak                      c) increasing    d) zero
- As a result of the process of electron capture ("K-capture") by  $^{211}\text{At}$ , the new isotope formed is:  
 a)  $^{210}\text{At}$                       b)  $^{212}\text{At}$                       c)  $^{211}\text{Po}$                       d)  $^{211}\text{Rn}$   
 e)  $^{207}\text{Bi}$
- Suppose a radioactive mother nucleus emits a  $\beta$ -particle. The mother and daughter nuclei are.....  
 a) isotones                      b) isobars                      c) isotop                      d) a,b&c  
 e) none
- Nonionizing radiations have wavelengths of  $\geq 10^{-7}$  m  
 a)  $10^{-10}$  km                      b)  $10^{-7}$  cm                      c)  $10^{-7}$  mm                      d)  $10^{-7}$  dm
- Binding energy per nucleon of helium is .....  
 a)  $45 \times 10^{-12} \mu\text{J}$                       b)  $1.125 \times 10^{-12} \text{ J}$                       c) 28.3 MeV  
 d) 28.3 MeV
- The energy equivalent of  $\alpha$ -particle is .....  
 a) 3.7 GeV                      b) 37 KJ                      c) 370 MeV                      d) 370 MeV
- ..... of energy is released during the  $\alpha$ -decay from  $^{238}_{92}\text{U}$  nucleus.
- The radioactive nuclei (parent nuclei) decay into a daughter nucleus &  $\alpha$ -particle. The  $\alpha$ -particle & daughter nucleus have K.E even if the parent nucleus is .....
- Whenever an  $\alpha$ -particle is emitted by a radioactive nucleus, the daughter nucleus must have a velocity in a direction opposite to that in which they ..... in ejected.
- Decay of  $\text{Bi}^{212}$  into thallium 208 with the emission of an alpha particle, the kinetic energy of the alpha particle .....
- Internal conversion coefficient is proportional the .....
- Shortest lived nuclides emit the least energetic alpha particle.  
 (True/False)

17. The  $\beta$ -particles emitted in radioactive decay of heavy nuclei are ordinary electrons. (True/False)
18. The energy of  $\beta$ -particle emitted in beta decay is very small ,usually less than 10Mev. (True/False)
19. The energy spectrum of  $\beta$ -particles has been studied with the help of a electric spectrometer. (True/False)
20. In the internal conversion process, a bound atomic electron is emitted. (True/False)

	BS Sem-VII/MSc Sem- III Final Term Exam.	Nuclear Phys-I(Phy-01701/21301) Max. Marks: 30 Time Allowed: 1hr 40 min	
Q#2			
I	Name the devices that depend upon ionization property.		2
II	What is the need of radiation detectors?		2
III	Can we accelerate neutrons by a cyclotron?		2
IV	What are the primary functions of (a) Electric field (b) Magnetic field in a cyclotron.		2
V	Can a cyclotron be used to accelerate electron? If not why?		2
VI	Between the Dee's of a cyclotron 1.5 meter in diameter an alternating potential difference of 15 mega cycles is applied. Calculate the energy in MeV of the protons issuing out of the cyclotron. Mass of proton = $1.672 \times 10^{-27}$ kg.		2
VII	An $\alpha$ - particle of energy 4.18 MeV is completely stopped in an ionization chamber. Calculate the pulse voltage and current in an external resistance of $10^6 \Omega$ . Energy required to produce one ion pair = 35 eV ; Capacitance of the chamber 25 $\mu\text{F}$ ?		2
Q#3			
I	Find the relation for maximum kinetic energy of a particle in cyclotron in terms of f.		4
II	Show that the betatron condition is $B = \frac{1}{2} B'$		4
III	A betatron working on an operating frequency of 60 Hertz has stable orbit of diameter 1.6 m. find the energy gained per turn as also final energy if the magnetic field at the orbit is 0.5 Tesla.		4
IV	(a)What is meant by the range of an alpha particle? Name the factors upon which it depends. (b) Explain straggling and its cause.		4